

**Outline of Mathematics**  
**May 20, 2013**

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**Note:** To look up references, see the Consciousness Bibliography, listing 10,000 books and articles, with full journal and author names, available in text and PDF file formats at [http://www.outline-of-knowledge.info/Consciousness\\_Bibliography/index.html](http://www.outline-of-knowledge.info/Consciousness_Bibliography/index.html).

**MATH>Mathematics**

**mathematics**

Mathematical ideas {mathematics} include intuition, empiricism, and abstraction; relations, functions, and transformations; logic, proof, and rigor; truth, certainty, and uncertainty; existence and quantity; and formalism, symbolic languages, and formal languages.

**mathematical objects**

In realism, mathematical objects can be abstract acausal things. In structuralism, they are structures and systems. In idealism, constructivism, and intuitionism, they are mental conventions. Perhaps, mathematics has no objects but is only analytic or linguistic conventions used by the mathematics community.

**mathematical operations**

Arithmetic main operations {arithmetic operation} {fundamental operation} are add, subtract, multiply, and divide. Division by 0 has no definition. Algebra operations {algebraic operation} include all arithmetic operations and their combinations. Exponential operations manipulate constant and variable powers to find exponents {evolution, arithmetic} and logarithms {involution, arithmetic}. Operations {trigonometric operation} can find sines, cosines, and tangents. Calculus operations {analytic operation} differentiate and integrate. Geometric operations {symmetry operation} are for transformation, translation, rotation, reflection, and inversion.

**mathematical foundations**

Mathematical objects, such as circles and tori, have reality or are concepts.

### **realism**

Realism says that object essences are real. Mathematical Realism, Idealism, and Platonism [Penrose, 2004] say mathematical objects are abstract essences: always existing, unchanging, timeless, without spatial location, sometimes with spatial extension, and metaphysical. They are not physical or mental. Mathematical objects are Ideals or Forms, fundamental reality that underlies physical and mental things. The physical follows mathematical laws and objects. The mental can comprehend mathematical laws and objects.

### **objectivism**

Objectivism says that people can know the real world. People can know things' essences and underlying reality.

### **language**

Because they share definitions and concepts, which are accessible reality or built by language, people can communicate about knowledge [Peirce, 1878].

### **positivism**

Positivism [Ayer, 1940] says that people can experience the real world, and can know what statements about it are right or wrong based on their and other people's measurements to determine corroborations or denials.

### **nominalism**

Nominalism says that object essences are only mental concepts, derived by human perception and reasoning from physical examples. Outside of language are only particulars. Conceptualism [Abelard, 1120] says that categories and rules are mental concepts shared by people that respond to similar world with similar minds, so universals are real insofar as they express similarities or essential object characteristics to which people respond to make concepts or dispositions. The physical world has no universals. Mathematical objects are human constructs, definitions, or concepts.

### **intuitionism**

Mathematical intuitionism [Brouwer, 1927] says that people, using cognitive skills and experience, develop understandings of mathematical ideas. Analysis of the real world reveals mathematical objects and concepts. People then describe mathematical objects and concepts using language and logic. People cannot know things' essences or if there is underlying reality.

### **constructivism**

Constructivism [Piaget, 1954] says that people use their, and other people's, innate and learned cognitive skills on perceptions to build concepts about their mental world, as opposed to discovering concepts of the real world. As opposed to just "telling", people use "showing" or proving [Wittgenstein, 1922]. Statements are not true but are provable or constructable. Statements are not false but have a counterexample.

### **formalism**

Formalism [Russell, 1919] says that mathematical laws and objects are mental logical/axiomatic systems. Logic and reasoning are unchanging, timeless, meta-mathematical, and abstract. Mathematical laws and objects can describe the physical.

## **MATH>Mathematics>Axiomatic Theory**

### **axiomatic theory**

Formal systems {axiomatic theory} can have primitives, definitions, axioms, and postulates in theory language {object language}.

### **primitive terms**

Object language has undefined primitive symbols or objects.

### **definitions**

Combining primitive terms defines further symbols or objects.

### **axioms**

Combining primitives and definitions can make assumptions. Axioms assume identity element existence, inverse element existence, commutation law, association law, and distribution law. Axioms are independent of other axioms.

### **postulates**

Object languages have valid statement structures and have logical rules for transforming statement structures. For example, variables can take values.

### **proofs**

Starting from primitive terms, definitions, axioms, and postulates, logic can prove theorem or formula statements, by deduction and formal proofs.

### **examples**

First-order predicate calculus {standard formalization} {first-order theory} is an example.

### **equivalences**

The same axiomatic theory can use different symbols and relations {formulation} {model, axiomatic theory}. Axiomatic-theory primitive terms and relations can have different meanings {interpretation, axiomatic theory} {representation, axiomatic theory}.

Axiomatic theories using different symbols and relations can be formally the same {isomorphism, axiomatic theory}. Axiomatic theories can always have isomorphic forms {representation theorem}. Categorical theory shows how to prove that two axiomatic theories are isomorphic.

### **consistency of axioms**

In axiomatic theories, contradiction means that statement and its inverse are true. If no theorem can contradict any axiom or theorem {consistency, mathematics}, statements and their inverses cannot both be true. Inconsistent theories have proofs that start with axiom or theorem and lead to inverse and so contradiction.

### **standard interpretation**

Formal-system symbols, words, axioms, coding, and rules have intended representation {standard interpretation}. If formal system is consistent, nonstandard interpretations exist that make statements that were false in standard interpretations true in nonstandard interpretations.

## **MATH>Mathematics>Axiomatic Theory>Decision Problem**

### **decision problem**

For axiomatic theories, does algorithm exist that can determine if formula is true or false {decision problem}? Do algorithms exist that can determine if formulas with no variables are true or false? Does showing that formula is true or false require formal proof?

### **Church thesis about truth**

In axiomatic theories that contain number theory, it is impossible to decide whether formula is true by any method except formal proof {Church's thesis, truth} {Church thesis, truth}.

## **MATH>Mathematics>Axiomatic Theory>Metamathematics**

### **metamathematics**

Logical-principle and simple formal-system theory {metamathematics} {proof theory, mathematics} can have no infinities, use minimum English, use existence theorems to show how to construct new objects, not use proof by contradiction, not use Zorn's lemma, and not use axiom of choice [Hilbert, 1899] [Kleene, 1952] [Tarski, 1983].

### **metalanguage**

Axiomatic-theory object languages can use higher-level natural-language terms {metalanguage} {syntax language}. Higher-level languages express theorems {metatheorem} about proofs, formal theories, languages, and logic [Hilbert, 1899] [Kleene, 1952] [Tarski, 1983]. Metamathematics is a metalanguage.

### **pure mathematics**

Mathematics can use only ideas of relation and class {logic of relations} {pure mathematics}.

## **MATH>Mathematics>Axiomatic Theory>Completeness**

### **completeness math**

Axiomatic theories can prove all theorems from terms and axioms {completeness, mathematics}. For example, propositional calculus and predicate calculus are complete. Incomplete theories cannot prove at least one true statement from terms and axioms.

### **d-completeness**

Complete theories can not allow new axioms {d-completeness}, because new axioms cause contradiction. For example, propositional calculus is d-complete. Predicate calculus is not d-complete, because it can add new axioms without contradicting any axiom or theorem and then use that axiom to prove more theorems.

## **Gödel completeness theorem**

Formal mathematical system, such as arithmetic, algebra, geometry, or set theory, has true propositions that people cannot prove true or false using theory definitions, axioms, rules, and theorems {completeness theorem} {Gödel completeness theorem} {Gödel's theorem} {incompleteness theorem}. Formal mathematical system that includes all true mathematics theorems cannot exist.

### **formal system**

Formal mathematical systems have finite numbers of word and symbol definitions, assumed-true axioms, and rules for combining words, symbols, and statements. Propositions combine words, symbols, and axioms. Typically, formal systems can form an infinite number of propositions.

For example, arithmetic is a formal mathematical system. It defines real numbers; sign symbols, such as plus and minus; operation symbols, such as addition and multiplication; grouping symbols, such as parentheses, brackets, commas, and spaces; variable symbols, such as  $x$  and  $y$ ; proposition and axiom symbols, such as  $A$  and  $B$ ; quantifier symbols, such as existential quantifier and universal quantifier; and logic symbols, such as AND, OR, NOT, IMPLIES, and IF. Arithmetic has axioms, such as commutation and association, about addition and multiplication operations. Arithmetic has axioms that are logic truths, such as "NOT of NOT of statement is statement", and "NOT of function existential quantifier is equivalent to universal quantifier for function NOT". Arithmetic has rules to derive propositions from previous propositions, for example, specifying variable value. Important rule is "If implication premise is true and implication is true, conclusion is true." Arithmetic can use word and symbol definitions, axioms, and rules to calculate and represent variables and functions.

### **proof**

Formal-mathematical-system purpose is to prove propositions true or false. Proofs are proposition series that use axioms, rules, and theorems to go from premise to conclusion. Valid formal mathematical systems prove true theorems and disprove false statements. Later proofs can use proved theorems.

### **natural number**

Natural numbers are finite digit sequences, with no signs or symbols. Natural numbers can have binary coding.

### **code array**

Because formal systems have finite parts and countable propositions, unique natural numbers can represent words, symbols, axioms, propositions, theorems, and rules. Proposition series can be unique natural-number series, and coded proofs are countable unique natural-number series. Listing all natural numbers represents the formal mathematical system. Number of binary digits needed to express all mathematical-system propositions and proofs is the same as number of propositions and proofs. Therefore, list length is the same as width. See Figure 1.

### **constructing new natural number**

Starting from unique natural-number list, one can construct a new natural number that is not in original system by the following method. Start with diagonal {diagonal slash} of the square list. See Figure 2.

Change marks at intersections of first row and column, second row and column, and so on. See Figure 3. New digit sequence has same length as the others and is unique natural number. It is not the same as any row or column sequence. It differs from first row and column at first number, differs from second row and column at second number, and so on.

### **completeness**

The unique natural number represents a statement that was not in original system. However, original list contained all system propositions. Therefore, no formal mathematical system can be complete. One can always derive new statements.

### **consistency**

Original list contained all proven system propositions, so new statement has no proof yet. New proposition is derivable from system but is neither true nor false yet. System plus new statement is indeterminate and inconsistent.

### **incompleteness or inconsistency**

If system lacks the proposition proof, proposition does not have correct form. However, proposition came directly from original propositions and so must have correct form. Therefore, statement is part of formal system but is indeterminate in trueness or falseness. Such formal systems are not consistent.

If system has the proposition proof, proposition is in current system but was not in original system. Such formal systems were incomplete.

If a formal system has only true propositions, and if the system does not prove the newly derived proposition, NOT of new proposition is false, because system already has all true-proposition proofs. New proposition is true, but its proof is not in system. Such systems are inconsistent.

If a formal system has only true propositions, and if the system does prove the newly derived proposition, new proposition is true, because all proven statements are true. However, all true propositions were already in system. Such systems were incomplete.

Formal systems must be either incomplete or inconsistent.

**question**

Number of binary digits needed to express all mathematical-system propositions and proofs is the same as number of propositions and proofs, so another binary digit must be added to all original propositions and to new proposition. To make a valid formal system, adding a proposition must change the original propositions. Note: Added binary digit can be the same as or different from added binary digit in original statements. Perhaps, the new system can be complete and consistent.

Figure 1

|   |   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | . | . | . |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | . | . | . |
| 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | . | . | . |
| 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | . | . | . |
| 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | . | . | . |
| 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | . | . | . |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | . | . | . |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | . | . | . |
| . | . | . | . | . | . | . | . | . | . | . |
| . | . | . | . | . | . | . | . | . | . | . |
| . | . | . | . | . | . | . | . | . | . | . |

Figure 2

|          |          |          |          |          |          |          |          |   |   |   |
|----------|----------|----------|----------|----------|----------|----------|----------|---|---|---|
| <b>1</b> | 1        | 1        | 1        | 1        | 1        | 1        | 1        | . | . | . |
| 0        | <b>0</b> | 0        | 0        | 0        | 0        | 0        | 0        | . | . | . |
| 1        | 0        | <b>1</b> | 0        | 1        | 0        | 1        | 0        | . | . | . |
| 0        | 1        | 0        | <b>1</b> | 0        | 1        | 0        | 1        | . | . | . |
| 1        | 0        | 0        | 1        | <b>0</b> | 0        | 1        | 0        | . | . | . |
| 0        | 1        | 0        | 0        | 1        | <b>0</b> | 0        | 1        | . | . | . |
| 0        | 0        | 1        | 0        | 0        | 1        | <b>0</b> | 0        | . | . | . |
| 1        | 0        | 0        | 0        | 1        | 0        | 0        | <b>0</b> | . | . | . |
| .        | .        | .        | .        | .        | .        | .        | .        | . | . | . |
| .        | .        | .        | .        | .        | .        | .        | .        | . | . | . |
| .        | .        | .        | .        | .        | .        | .        | .        | . | . | . |

Figure 3

from Figure 2 diagonal

1 0 1 1 0 0 0 0 . . .

to new sequence by changing each position

0 1 0 0 1 1 1 1 . . .

### **omega consistency**

Perhaps, formal system can have proofs for all true statements {omega consistency, completeness}. However, the incompleteness theorem demonstrates that consistent formal system has at least one true statement that has no proof and so is incomplete, as shown by Gödel. Incompleteness theorem shows that the proposition that formal system is omega consistent is not provable by the formal system. For example, logic and number theory have no proof that they are consistent using formal number theory. Therefore, information derivable from formal systems has limits. No axiom set is sufficient to prove all arithmetic or mathematics, by Gödel's proof.

## **MATH>Mathematics>Axiomatic Theory>Postulates**

### **induction postulate**

If "one" has a property, and number successors have the property, all numbers have the property {mathematical induction postulate} {induction postulate}. It is a Dedekind-Peano postulate.

### **Peano postulates**

Positive-integer axiomatic theory has postulates {Peano's postulates} {Peano postulates} {Dedekind-Peano postulates}. "One" is a number. The number "one" is not any number's successor. A number successor is a number. No two numbers have same successor. If "one" has property and if number successor has property, all numbers have property {induction postulate, Peano}.

## **MATH>Mathematics>Axiomatic Theory>Kinds**

### **geometry theory**

Geometry is an axiomatic system {geometry theory}.

#### **primitives**

Geometry can use two undefined ideas: point and order.

#### **symbols**

Geometry can use three symbols: point or line, segment, and motion or congruence. Points can be ordered-pairs. Line can be a ratio. Congruence can be translation, rotation, and reflection.

#### **axioms**

Points can form a set. At least one point exists. If there is a point, then there is another point. Given two points, there are lines. Given a line, there is another line. Given two lines, there is a space.

#### **postulates**

Axiomatized geometry has five postulates, similar to Euclid's five postulates. Lines can have no multiple points {Jordan curve, geometry}. Figures can have infinite perimeters.

In three-dimensional space, continuity axiom is true, but planes and surfaces do not need this axiom.

Dimension number is not necessarily coordinate number or point multiplicity.

#### **theory**

Geometry uses point, line, plane, how point "lies on" line, how point "lies on" plane, how point pairs are congruent, how angles are congruent, and/or how points order on lines {betweenness, geometry}.

#### **consistency**

Geometry is consistent if arithmetic is consistent.

### **natural number theory**

Counting numbers, and all numbers, can form an axiomatic theory {natural number theory}.

#### **axioms**

Start with null set and with two axioms. Numbers correspond to two sets of previously constructed numbers, with no member of left set greater than or equal to members of right set. Number is less than or equal to another number, if and only if no member of first-number left set is greater than or equal to second number and no member of second-number right set is less than or equal to first number.

#### **axioms: procedure**

First, make null set both right and left set, to obtain number zero. Then make right set null and left set contain number zero to obtain number one. Continue to build all counting numbers. All other numbers can derive from counting numbers.

#### **equivalence**

Equivalence or one-to-one correspondence can construct counting numbers. Start with null set. Define zero as cardinal number of elements in null set and equivalent sets. Define one as number of elements in set that contains only zero as element. Define two as number of elements in set that contains only elements zero and one. Define N as number of elements in set that contains only elements zero through N.

### **numbers in general**

Axiomatizing natural numbers allows axiomatizing all number types. The number "one" belongs to a set. Set members have one and only one successor. If two successors are equal, then members are equal. The number "one" is not any number's successor. If subset contains "one" and another number, then number successor is in subset and subset is same as whole set.

### **integers**

Positive integers can be an axiomatic system. Undefined terms are "one", "number", and "successor of". Dedekind-Peano postulates can construct the positive integers.

### **non-Archimedean geometry**

Geometries {non-Archimedean geometry} that violate Archimedes axiom can be axiomatic systems.

## **MATH>Mathematics>History**

### **Pierre de Fermat [Fermat, Pierre de]**

mathematician

France

1629 to 1640

He lived 1601 to 1665, invented Fermat's minor theorem and Fermat's last theorem [1640], and studied differential calculus, maxima/minima in war and astronomy [1629], curve and surface tangents and normals in optics and motion, and infinite-descent method.

### **Jacob Bernoulli [Bernoulli, Jacob]**

mathematician

Basel, Switzerland

1686

Art of Conjecturing [1686]

He lived 1654 to 1705 and studied calculus of variations, variable separation, lemniscate curve, differential equations, and inflection points. He invented Bernoulli's theorem and Bernoulli equation. If event can have two outcomes, each with probability, after many independent events, relative frequency approaches probability {weak law of large numbers, Bernoulli}. Perhaps, probabilities are inferable from frequencies.

### **Johann Bernoulli [Bernoulli, Johann]**

mathematician

Basel, Switzerland

1691

He lived 1667 to 1748 and studied series, arithmetic series, geometric series, differential equations [1691], astroid [1691], curvature radius, Bernoulli numbers, and curve rectification. Wire shape allows bead to slide from one end to the other in shortest possible time {brachistochrone, Bernoulli} [1696].

### **Leonhard Euler [Euler, Leonhard]**

mathematician

Basel, Switzerland

1748 to 1770

Introduction to Infinite Analysis [1748]; Institutes of Integral Calculus [1770]; Institutes of Differential Calculus [1770]

He lived 1707 to 1783 and invented Euler's formula, Euler number, and Euler constant. He studied incompressible non-rotating non-viscous fluid flows {potential flow}. He studied non-homogeneous nth order differential equations, partial-fractions method, explicit and implicit functions, networks, harmonic and divergent series, hypergeometric functions, natural logarithms, partial derivatives, multiple integrals, calculus of variations, finite-differences method, and gamma and beta functions.

First-order equations can be exact differentials. Newton's laws can depend on a maximizing-minimizing principle {principle of stationary action} {stationary action principle} {Euler-Lagrange equations}.

### **European 1750**

mathematician

Europe

1750

European mathematicians used induction, generalization, agreement method, difference method, analogy, and causal generalization.

### **Adrien-Marie Legendre [Legendre, Adrien-Marie]**

mathematician

Paris, France

1794 to 1830

Elements of Geometry [1794]; Theory of Numbers [1808]; Exercises in Integral Calculus [1811 to 1830]

He lived 1752 to 1833, studied number theory and elliptic integrals, and invented Legendre function and Legendre differential equation.

### **European 1800**

mathematician

Europe

1800

European mathematicians studied populations, statistics, distributions, sampling, correlation, and correlation coefficient.

### **Karl Friedrich Gauss [Gauss, Karl Friedrich]**

mathematician

Göttingen, Germany

1801 to 1827

Disquisitiones on Arithmetic [1801]; General Investigations of Curved Surfaces [1827]

He lived 1777 to 1855 and studied Earth magnetic field. In statistics, he developed Gaussian distribution {normal distribution, Gauss}, variance, standard deviation, mean standard error, least-squares method, and regression. In number theory, he worked on analytic number theory, algebraic numbers, complex numbers, hypercomplex numbers, Diophantine analysis, and theory of forms. In geometry, he invented seventeen-sided regular polygons, used substitute parallel axiom for non-Euclidean geometry, and studied curvature, congruence theory, and Gaussian coordinates. In algebra, he invented fundamental theorem of algebra and studied elliptic functions, Gauss characteristic equation, and central limit theorem. In vector theory, he worked with dot product and cross product. In physics, he developed dynamic equations that minimized quantity and Principle of Least Constraint.

### **Niels Abel [Abel, Niels]**

mathematician

Norway

1824 to 1826

He lived 1802 to 1829 and invented elliptic-function addition theorems and integrals. He studied quintic polynomials [1824], elliptic functions, series, fields, and rings. He invented Abelian integrals [1826], Abel's theorem, Abel summability, and Abelian group or commutative group.

### **Lejeune Dirichlet [Dirichlet, Lejeune]**

mathematician

Belgium

1825 to 1839

Lectures on Number Theory [1839]

He lived 1805 to 1859 and invented Dirichlet series, Dirichlet conditions, and Dirichlet principle or Thomson principle. He studied analytic number theory [1825].

### **European 1830**

mathematician  
Europe  
1830

European mathematicians studied transitivity, reflexivity, symmetry, equivalence, axial symmetry, radial symmetry, rotation, reflection, translation, and inversion.

**Nikolai Ivanovich Lobachevski [Lobachevski, Nikolai Ivanovich]**

mathematician  
Kazan, Russia  
1830

On the Principles of Geometry [1830]

He lived 1793 to 1856 and invented Lobachevsky's rule. He used substitute parallel axiom, applied to two boundary lines with angle to the perpendicular, to make non-Euclidean hyperbolic geometry {Lobachevskian geometry, Lobachevski}. People do not know Euclid's axioms with certainty, and they are not true a priori.

**Leopold Kronecker [Kronecker, Leopold]**

mathematician  
Germany  
1845 to 1887

On Complex Units [1845]; Foundation of an Arithmetic Theory of General Algebra [1882]; On the Concept of Numbers [1887]

He lived 1823 to 1891 and helped develop intuitionism. He invented Kronecker delta function and studied fields [1881].

**Karl Weierstrass [Weierstrass, Karl]**

mathematician  
Germany  
1854

On the Theory of Abelian Functions [1854]

He lived 1815 to 1897, used arithmetic concepts for mathematical analysis, and studied real number theory, analytic and elliptic functions, and uniform convergence. He invented Weierstrass-Bolzano theorem [1854]. To remove contradictions introduced by infinitesimals, he reformulated calculus using limits and exhaustion method. Elliptic complex functions are sums of convergent power functions. Irrational numbers are rational-number-series convergences.

**Felix Klein [Klein, Felix]**

mathematician  
Germany  
1871

On the So-called Non-Euclidean Geometry [1871]; Erlanger Program [1872: Erlanger is town in Germany]; Riemann's Theory of Algebraic Functions and their Integrals [1882]

He lived 1849 to 1925 and set forth Erlangen program [1872]. He invented Klein's bottle and metric. In three dimensions, all metric geometries are projective geometry augmented by a quadric {absolute, geometry} or a curve related to absolute.

**Ferdinand Georg Frobenius [Frobenius, Ferdinand Georg]**

mathematician  
Germany  
1878

He lived 1849 to 1917 and studied linear algebra [1878], series, and groups.

**Hermann Amandus Schwarz [Schwarz, Hermann Amandus]**

mathematician  
Germany  
1885

He lived 1843 to 1921 and invented Schwarz statistics criterion, Schwarz's inequality [1885], and Schwarz's paradox.

### **Émile Borel [Borel, Émile]**

mathematician

Paris, France

1895 to 1946

Risk [1913]; Space and time [1921]; Treatise on calculation of probability and its applications [1924 to 1934]; Paradoxes of infinity [1946]

He lived 1871 to 1956 and studied functions using series and measure theory [1895], invented Heine-Borel theorem, and helped develop intuitionism.

### **David Hilbert [Hilbert, David]**

mathematician

Germany

1897 to 1912

Report on Numbers [1897]; Foundations of Geometry [1899]; 23 Unsolved Problems of Mathematics [1900: at International Congress of Mathematicians, Paris]; Elements and Principles of Mathematics [1912]

He lived 1862 to 1943 and studied formal systems, proof theory, metamathematics, and Erlanger Program. He studied real numbers using connection, calculation, order, and continuity axioms. He invented Hilbert space and Hilbert-Schmidt theorem. He posed problems {Hilbert program} for 20th century mathematicians to solve [1900]. His tenth problem {Entscheidungsproblem} asked if theorem-proving algorithms are possible. Integral equations and complete orthogonal-system theories relate.

### **Epistemology**

Mathematics can depend on proofs using symbol language {formalism, Hilbert}. Mathematics branches can be formal and studied at higher level {metamathematics, Hilbert}, but do not need infinitely high level. Meaningful mathematics is about finite objects and relations. The infinite hotel {Hilbert hotel} has an infinite number of rooms, so it has infinitely many vacancies, no matter how many people.

### **European 1900**

mathematician

Europe

1900

European mathematicians studied topology or analysis situs, as in neighborhood, completeness, compactness, connectedness, winding number, homeomorphy, Königsberg bridges problem, four-color theorem or map problem, manifold, simplex, and tessellation. They studied impredicative definition, as in Burali-Forti paradox, barber paradox, Richard's paradox, heterological paradox, Russell's paradox, hangman's paradox, and Newcomb's problem.

### **Henri Lebesgue [Lebesgue, Henri]**

mathematician

Paris, France

1901

On a generalization of the definite integral [1901]

He lived 1875 to 1941 and invented Lebesgue integral and Riemann-Lebesgue lemma, studied measure theory [1901], and helped develop intuitionism.

### **Jacques Hadamard [Hadamard, Jacques]**

mathematician

France

1903 to 1945

Mathematician's Mind or Psychology of Invention in the Mathematical Field [1945]

He lived 1865 to 1963 and studied functionals [1903], characteristic equations and helped develop intuitionism.

### **Elie Cartan [Cartan, Elie]**

mathematician

France

1904 to 1945

Exterior differential systems and their geometric applications [1945]

He lived 1869 to 1951 and studied hypercomplex numbers, Lie group theory, differential geometry [1904], and exterior derivatives.

### **Jules Henri Poincaré [Poincaré, Jules Henri]**

mathematician/philosopher

Paris, France

1905 to 1908

Science and Hypothesis [1905]; Foundations of Science [1908]

He lived 1854 to 1912, helped develop intuitionism, and studied function theory, differential equations, orbits, and combinatorial topology. He found special-relativity equations [1905]. He showed how to keep distances constant as observed from different constant motions in flat space-time {Poincaré motion} {inhomogeneous Lorentz motion}, by lengthening light-cone along space dimensions and shrinking light-cone along time dimension. After systems reach largest phase-space region, they can return to all smaller regions over times much longer than universe age {Poincaré recurrence}.

#### **Epistemology**

Mathematical thinking is purely mental and so can reveal what is essential in mind. Unconscious thinking has preceded insight. Mind unconsciously selects possible solutions using innate or consciously formulated rules. Thinking appears to move in one direction and has purpose. Aesthetic value is an important creativity component.

Thinking converges on truth, but absolute truth is unattainable. Statement is possibly true if it is not necessary that it is not true. Contradictions are necessarily not true. Statements that do not involve contradiction state logical possibility. Not all contradictions are apparent. Nature contains contradictions, so contradictions can state possibilities.

Science decides what is naturally possible and naturally impossible. Epistemic possibility is what is consistent with human knowledge states. Possible truth is true in at least one possible world. Necessary truth is true in all possible worlds. Possibility and necessity are arbitrary rules about word use. Concept meaning depends on possible and impossible.

Definition can quantify over all class objects {vicious-circle principle, Poincaré} {impredicative definition, Poincaré} or not include them {predicative definition, Poincaré}.

### **George David Birkhoff [Birkhoff, George David]**

mathematician

USA

1907 to 1933

Asymptotic Properties of Certain Ordinary Differential Equations with Applications to Boundary Value and Expansion Problems [1907]; Relativity and Modern Physics [1923: with R. E. Langer]; Aesthetic Measure [1933]

He lived 1884 to 1944, invented Birkhoff's theorem [1909], proved Poincaré's Last Geometric Theorem [1913], discovered ergodic theorem [1931 to 1932], studied asymptotic series, and helped develop quantum logic.

### **Hermann Weyl [Weyl, Hermann]**

mathematician

Germany/USA

1918 to 1952

Continuum [1918]; Symmetry [1952]

He lived 1885 to 1955, studied integral equations, helped develop intuitionism, and studied universe symmetries. Abstract objects exist only if they have predicative definitions {predicative theory, Weyl}. Predicative definition must be countable.

### **European 1920**

mathematician

Europe

1920

European mathematicians invented Students' t distribution, F distribution, chi-square distribution, variation coefficient, factor analysis, sequential analysis, estimation, unbiased estimate, and confidence interval.

### **Nicholas Bourbaki [Bourbaki, Nicholas]**

mathematics group

France

1939

Elements of Mathematics [1939]

Mathematicians, including Claude Chevalley, André Weil, Henri Cartan, and Jean Dieudonné, studied modern-mathematics foundations.

### **European 1940**

mathematician

Europe

1940

European mathematicians studied operations research, linear programming, simplex method, marginal value principle, formal languages, algorithms, and decision problem.

### **European 1960**

mathematician

Europe

1960

European mathematicians studied coding, check bits, parity checking, logical sum checking, weighted check sums, modulo 37 with progressive digitizing, Hamming code, geometric code, variable length code, instantaneous code, Huffman code, compression, hashing, and Gray code [Hamming, 1960].

### **Imre Lakatos [Lakatos, Imre]**

mathematician

Hungary/England

1963 to 1976

Proofs and Refutations [1963]

He lived 1922 to 1974. He founded empirical mathematics philosophy, in which people can know truth by Methodology of Scientific Research Programmes. He opposed the philosopher of science Paul Feyerabend.

### **Alexandr D. Alexandrov [Alexandrov, Alexandr D.]/Andrei N. Kolmogorov [Kolmogorov, Andrei N.]/Mikhail A. Lavrent'ev [Lavrent'ev, Mikhail A.]**

mathematician

Russia

1984

Mathematics: its content, methods, and meaning [1984: translated by S. H. Gould and T. Bartha]

Kolmogorov lived 1903 to 1987 and developed measure theory [1965].

### **James Gleick [Gleick, James]**

writer

USA

1987

Chaos [1987]

He wrote popular science.

### **Reuben Hersh [Hersh, Reuben]**

mathematician

USA

1997

What Is Mathematics Really? [1997]

He wrote popular science.

## **MATH>Mathematics>History>Algebra**

### **algebra invented**

mathematician

Babylonia/Egypt

-1950 to -1750

algebra

Middle Eastern mathematicians solved general linear and quadratic equations using variables.

**Mohammad ibn Musa al-Khwarizmi [al-Khwarizmi, Mohammad ibn Musa] or Algorizm or Muhammad Bin Musa al-Khwarizmi [al-Khwarizmi, Muhammad Bin Musa]**

mathematician

Baghdad, Iraq

825

Arithmetic [825]; Algebra [839]; world map [830]

He lived 770 to 840, used Hindu numbers and fractions, and studied algebra. Adelard of Bath and Gerard of Cremona [1100 to 1150] translated his works and so transferred Indian and Islamic philosophy to Europe.

**al-Karkhi or al-Karaji**

mathematician

Baghdad, Iraq

1010

Glorious on Algebra [1010]

He lived 953 to 1029 and invented completing the square.

**Girolamo Cardano [Cardano, Girolamo] or Gerolamo Cardano [Cardano, Gerolamo]**

mathematician/physician/inventor

Milan, Italy

1524 to 1545

Book of Games of Chance [1524]; Practice of Arithmetic and Simple Mensuration [1540]; Great Arts [1545]; universal joint or cardan shaft [1545]; combination lock

He lived 1501 to 1576 and found general-cubic-equation and general-quartic-equation solutions. He studied negative-number square roots and essentially discovered complex numbers, finding complex-number roots of  $x + y = 10$  and  $x*y = 40$ . Cubic equation can be  $x^3 = p*x + q$ , where  $p$  and  $q$  can be zero, positive, or negative. Cubic equation can have up to three roots, all real numbers. If there are three roots {casus irreducibilis}, intermediate steps to solution can require complex numbers. He also studied game probability and began probability theory.

**Politics**

Culture and politics relate, as actual states and history show.

**Nicolo Tartaglia [Tartaglia, Nicolo]**

mathematician

Venice, Italy

1537 to 1546

New Sciences [1537]; New Problems and Inventions [1546]

He lived 1499 to 1557 and found general solution to cubic equation.

**Lodovico Ferrari [Ferrari, Lodovico]**

mathematician

Italy

1542

He lived 1522 to 1565 and found general solution to quartic equation [1542].

**Thomas Harriott [Harriott, Thomas]**

mathematician

England

1588

Brief and True Report of the New Found Land of Virginia [1588: close-packing]

He lived 1560 to 1621, used modern algebra notation, and studied trajectories. He invented refraction law.

**Gabriel Cramer [Cramer, Gabriel]**

mathematician  
Geneva, Switzerland  
1749  
Introduction to the analysis of algebraic curved lines [1749]  
He lived 1704 to 1752 and invented Cramer's rule.

**Alexandre-Theophile Vandermonde [Vandermonde, Alexandre-Theophile]**

mathematician  
Alsace  
1771  
Memoir of equation solving [1771]; Remarks on the problems of position [1771: about knots]  
He lived 1735 to 1796 and invented determinant minor.

**George Peacock [Peacock, George]**

mathematician  
London, England  
1830  
Treatise on Algebra [1830]  
He lived 1791 to 1858 and studied algebra systems and permanence of form.

**Augustus De Morgan [De Morgan, Augustus]**

mathematician  
England  
1830 to 1849  
Elements of Arithmetic [1830]; Induction [1838]; Formal Logic [1847]; Trigonometry and Double Algebra [1849]  
He lived 1806 to 1871 and studied divergent series. He invented De Morgan's laws [1849] of algebra of classes: commutation, association, inverse, identity, distribution, and null.

**Ernst Steinitz [Steinitz, Ernst]**

mathematician  
Germany  
1910  
Algebraic Theory of Fields [1910]  
He lived 1871 to 1928 and studied algebraic field theory.

**Georg Pólya [Pólya, Georg] or George Pólya [Pólya, George]**

mathematician  
USA  
1954 to 1957  
How to solve it [1945]; Mathematics and Plausible Reasoning [1954]  
He lived 1887 to 1985 and studied problem solving and problem-solving heuristics and invented counting formula. Plausibility depends on authority or reliability of information source used to justify proposition, not on probabilities of alternatives. Reasoning-chain plausibility is least-plausible-proposition plausibility.

**Mikio Sato [Sato, Mikio]**

mathematician  
Japan  
1958  
He lived 1928 to ? and studied hyperfunctions [1958].

**MATH>Mathematics>History>Analysis**

**Madhava of Sangamagramma**

mathematician  
Kerala, India  
1380 to 1410

Rig-veda comments [1380 to 1410]

He lived 1350 to 1425, founded Kerala School of mathematics, developed infinite series, and started mathematical analysis.

**Augustin-Louis Cauchy [Cauchy, Augustin-Louis]**

mathematician

Paris, France

1814 to 1829

Lessons on Differential Calculus [1829]

He lived 1789 to 1857, used arithmetic concepts for mathematical analysis, and began complex-variable function theory [1814]. He invented Cauchy's principle, Cauchy convergence criterion, and Cauchy integral theorem. He studied method of characteristics, theory of content, and spaces. Separating first-order partial-differential-equation variables can make ordinary-differential-equation systems. First-order partial differential equation systems can describe elastic-media properties.

**Friedrich Bessel [Bessel, Friedrich]**

mathematician

Germany

1817 to 1824

He lived 1784 to 1846 and invented Bessel equation [1817 to 1824] and Bessel's inequality.

**Carl Gustav Jacob Jacobi [Jacobi, Carl Gustav Jacob]**

mathematician

Germany

1829 to 1841

Fundamental new theory of elliptical functions [1829]; On determinants of functions [1841]

He lived 1804 to 1851 and studied elliptic integrals, function theory, and inverse elliptic functions {theta function, Jacobi}. He invented Jacobian.

**Christoph Gudermann [Gudermann, Christoph]**

mathematician

Munster, Germany

1840 to 1841

He lived 1798 to 1852 and worked with elliptic functions [1840 to 1841]. Elliptic functions are sums of converging power terms.

**Charles Hermite [Hermite, Charles]**

mathematician

France

1858 to 1864

On a new development in function series [1864]

He lived 1822 to 1901 and invented Hermitean operators and Hermite functions [1858 to 1864].

**Alfred Clebsch [Clebsch, Alfred]**

mathematician

Germany

1862 to 1866

Theory of Elasticity in Fields [1862]; On the Applications of Abelian Functions in Geometry [1864]; Theory of Abelian Functions [1866: with Paul Gordan]

He lived 1833 to 1872 and studied genus of curves {Clebsch-Gordan coefficients}.

**Paul Gordan [Gordan, Paul]**

mathematician

Germany

1866

Theory of Abelian Functions [1866: with Alfred Clebsch]

He lived 1837 to 1912.

**William Kingdon Clifford [Clifford, William Kingdon]**

mathematician/philosopher

England

1874 to 1877

Body and Mind [1874]; Ethics of Belief [1877]; Applications of Grassmann's extensive algebra [1878]

He lived 1845 to 1879 and invented geometric product and Clifford algebras [1878]. He studied complex analysis.

Addition does not necessarily combine two units of same kind but instead defines relations, as in complex numbers or hypernumbers. People have innate learning, which developed through evolution {evolutionary epistemology, Clifford}.

Mind grows by evolution {creative evolution}.

**Sonja Kowalewski [Kowalewski, Sonja] or Sophie Kowalewski [Kowalewski, Sophie] or Sofia Kovalevskia [Kovalevskia, Sofia]**

mathematician

Russia

1889

On the problem of rotation of solid body around fixed point [1889]

She lived 1850 to 1891 and studied elliptic functions and power-series sums.

**Stefan Banach [Banach, Stefan]**

mathematician

Poland

1922

He lived 1892 to 1945 and studied functional analysis, projection theorem, triangle inequality, and adjoints. He invented Banach spaces, Hahn-Banach theorem [1922], and Banach algebra.

**Hans Hahn [Hahn, Hans]**

mathematician

Austria

1922

He lived 1879 to 1934 and invented Hahn-Banach theorem [1922].

**Sergei Lvovich Sobolev [Sobolev, Sergei Lvovich]**

mathematician

Russia

1932 to 1939

He lived 1920 to 1990 and developed generalized-function spaces [1932].

**Richard von Mises [Mises, Richard von]**

physicist

Ukraine/USA

1941

He lived 1883 to 1953 and developed measure theory [1941].

**Laurent Schwartz [Schwartz, Laurent]**

mathematician

Germany

1945 to 1950

He lived 1915 to 2002, developed distribution theory {theory of distributions}, and developed generalized-function theory, allowing discontinuous-function derivatives [1945 to 1950].

**MATH>Mathematics>History>Ancient Mathematics**

**Mesopotamian calendar**

calendar

Mesopotamia  
-4000  
Mesopotamian calendar [-4000: lunar calendar]  
It had 12 months and predicted yearly flooding.

### **Egyptian calendar**

calendar  
Egypt  
-2770  
Egyptian calendar [-2770: solar calendar]  
Year was 365 days long, with 12 months of 30 days. Calendar predicted Nile flooding.

### **Sun and Moon position**

mathematician  
Babylonia  
-1750  
Sun and Moon position prediction  
Babylonian mathematicians predicted Sun and Moon positions.

### **Ahmes**

mathematician  
Fayum, Egypt  
-1650  
Rhind papyrus or Ahmes Papyrus [-1650]  
He lived -1680 to -1620, solved practical architecture problems, calculated astronomical events, and used simple interest, compound interest, principal, and rate. Multiplication is repeated doubling, and division is repeated halving.

### **Pythagoras**

mathematician  
Greece  
-530  
He lived -580 or -569 to -500 and invented gnomon and Pythagorean theorem. He used similar figures, proportions, Pythagorean triples, Golden Ratio, Golden Section, and Golden Rectangle, and triangular, square, perfect, amicable, and prime numbers.

### **sexagesimal number system**

mathematician  
Babylonia  
-500  
sexagesimal number system  
Babylonian mathematicians used number system based on 60 {sexagesimal number system}. They predicted Sun, Moon, and planet positions based on previous positions that they had recorded.

### **Platonists**

mathematics school  
Greece  
-400 to -350  
Platonists used inference, proof, deduction, and induction. They studied regular polyhedra, conic sections, prism, pyramid, cone, cylinder, perimeter, area, volume, and surfaces. Regular polyhedra are tetrahedron, icosahedron, and dodecahedron.

### **Platonists**

mathematics school  
Greece  
-300 to -150

Platonists studied prisms, pyramids, cylinders, cones, conic sections, and the five regular polyhedra. They gave circle 360 degrees. They assumed proposition is true and then deduced consequences, until statement is clearly true or false {method of analysis}. They assumed theorem and showed that theorem leads to contradiction, so theorem is false {reductio ad absurdum, Platonists}. They used deductive proof {deduction, Platonists}.

**Surya Siddhanta**

mathematician  
India  
400  
Sun Principles [400: about astronomical calculations]  
It depends on Persian books.

**Thabit or Tabit ibn Qorra or al-Sabi Thabit ibn Qurra al-Harrani [Thabit ibn Qurra al-Harrani, al-Sabi]**

mathematician  
Baghdad, Iraq  
870  
Conics [870]  
He lived 836 to 901.

**al-Battani or al-Batin or Albategnius or Albategni or Albatenius**

mathematician  
Baghdad, Iraq  
900  
Book of Astronomical Tables [900: about astronomical calculations]  
He lived 868 to 929 and found ecliptic angle and solar-year length.

**al-Hazen or Alhazen or Ibn al-Haytham [al-Haytham, Ibn] or Abu Ali al-Hassan**

mathematician/physicist  
Baghdad, Iraq/Egypt  
1030  
Treasury of Optics [1030]  
He lived 965 to 1039 and studied perspective, projection, vanishing points, and cubic equations.

**Bhaskara or Bhaskara II or Bhaskaracharya**

mathematician  
Ujjain, Mahdya Pradesh, India  
1114 to 1140  
Sun Spheres and Light [1114: spheres, planets, and decimal number system]; Diadem of an Astronomical System [1140]  
He lived 1114 to 1185, followed Brahmagupta, and used combinations and permutations.

**Jamshid al-Kashi [al-Kashi, Jamshid]**

mathematician  
Samarkand, Kazakhstan  
1407 to 1427  
Stairway of Heaven [1407]; Compendium of the Science of Astronomy [1411]; Key to Arithmetic [1427]  
He lived 1390 to 1450 and used base-ten number system, decimals, and negative powers.

**MATH>Mathematics>History>Axiomatic Theory**

**Moritz Pasch [Pasch, Moritz]**

mathematician  
Germany  
1882  
He lived 1843 to 1930 and studied geometry foundations [1882], especially line and point interchangeability.

**Giuseppe Peano [Peano, Giuseppe]**

mathematician

Turin, Italy

1890

Mathematical Formulas [1890]

He lived 1858 to 1932. He invented logical notation, which Russell used. He studied axiomatic number systems. He invented Peano's postulates about rational numbers, based on Dedekind's work. He used reflexive, symmetric, and transitive axioms to derive rational numbers from natural numbers.

**Oswald Veblen [Veblen, Oswald]**

mathematician

England

1910 to 1918

Projective Geometry [1910 to 1918: with John Wesley Young, 2 volumes]

He lived 1880 to 1960 and axiomatized geometry using ideas of point and order.

**Gino Fano [Fano, Gino]**

mathematician

Turin, Italy

1914 to 1930

Lessons in Descriptive Geometry [1914]; Lessons in Analytical and Projective Geometry [1930: with Alessandro Terracini]

He lived 1871 to 1952 and invented line and space axiomatic systems, building from points to lines to space. The three complete-quadrilateral diagonal points are never collinear {Fano's axiom}.

**Kurt Gödel [Gödel, Kurt]**

mathematician/logician

Czech Republic/Slovakia/USA

1930 to 1939

On Formally Undecidable Propositions of Principia Mathematica and related systems [1931]; Consistency of the Axiom of Choice and of the Generalized Continuum-hypothesis with the Axioms of Set Theory [1940]

He lived 1906 to 1978. First-order predicate calculus and first-order logic are complete [1930]. All formal arithmetic systems must be incomplete [1931]. For all formal and consistent arithmetic systems, at least one true arithmetic proposition cannot be formally decidable. Neither proposition nor negation has proof, so arithmetic system is incomplete {Gödel's first incompleteness theorem}. Propositions are statements about numbers. Propositions have Gödel-number codes. Systems have propositions about propositions, and at least one such statement is not provable, because proofs use self-referential number statements. Therefore, it is impossible to prove system consistency using arithmetic.

Formal or logical systems are logically equivalent to recursively definable functions and arithmetic systems. Computing machines embody such functions. Therefore, machines can never prove their consistency or completeness.

The continuum hypothesis is consistent with basic set-theory axioms [1938 to 1939].

**Epistemology**

Definitions can specify class elements and their relations, and relations can make new elements {recursive definition}.

Mathematical objects and concepts are real and separate from mind. People know fundamental mathematical truths by intuition.

**Haskell Curry [Curry, Haskell]**

mathematician

England

1951

Outlines of a Formalist Philosophy of Mathematics [1951]

He lived 1900 to 1982. Mathematics branches become more formal over time, until they are deductive systems. Mathematics is about deductive systems.

**William Craig [Craig, William]**

mathematician

USA

1953

On Axiomatizability within a System [1953]

For axiomatic theories, subsets can use only some original terms but contain same theorems {Craig's theorem}.

## **MATH>Mathematics>History>Calculus**

### **Takakazu Seki [Seki, Takakazu] or Kowa Seki [Seki, Kowa] or Seki Kowa [Seki, Kowa]**

mathematician

Edo (Tokyo), Japan

1674 to 1683

Mathematical Methods for Finding Details [1674]

He lived 1642 to 1708, invented calculus, and used determinants [1683]. Japanese temple geometry flourished at this time.

### **Michel Rolle [Rolle, Michel]**

mathematician

France

1691

He lived 1652 to 1719 and invented Rolle's theorem [1691].

### **Daniel Bernoulli [Bernoulli, Daniel]**

mathematician

Basel, Switzerland

1734

Hydrodynamics [1734]

He lived 1700 to 1782. He solved differential equations by isolating variables. He developed cylindrical and spherical wave equations to represent organ-pipe sounds. He invented vibrating string equation. He studied hydrodynamics and invented Bernoulli's law [1734].

### **Pierre-Louis Moreau de Maupertuis [Maupertuis, Pierre-Louis Moreau de]**

mathematician

France

1744 to 1746

He lived 1698 to 1759 and developed dynamics maximizing-minimizing principle (principle of least action or least-action principle or principle of stationary action or stationary-action principle).

### **Joseph Louis Lagrange [Lagrange, Joseph Louis]**

mathematician

Paris, France/Italy

1771 to 1811

Turin Miscellany [1771]; Analytical Mechanics [1788 and 1811]

He lived 1736 to 1813 and studied calculus of variations, mean-value theorem, spherical coordinates, solution envelopes, adjoint equations, finite-differences method, and perturbation methods. He solved differential-equation systems using conic-section deviations. Newton's laws can depend on principle of stationary action in Euler-Lagrange equations. Natural numbers are sums of four natural-number squares.

### **Jean Le Rond d'Alembert [d'Alembert, Jean Le Rond]**

mathematician

Paris, France

1772

Preliminary Discourse to the Encyclopedia [1772]

He lived 1717 to 1783 and studied differential equations and multiple integrals and invented d'Alembert's test. He found that Newton's 3rd law applies to free bodies {d'Alembert's principle}.

**Pierre-Simon de Laplace [Laplace, Pierre-Simon de]**

mathematician  
 Paris, France  
 1780

Mémoire sur la probabilité des causes par les événements or Memoir on the probability of events [1774]; Théorie du mouvement et de la figure elliptique des planètes or Theory of movement of planet elliptical orbits [1776]; Mécanique céleste or Celestial Mechanics [1780]; Essai philosophique sur les probabilités or Philosophical essay on probabilities [1814]

He lived 1749 to 1827 and studied partial differential equations, Laplace transforms and operators, perturbations method, spherical coordinates, finite-differences method, and divergence theorem.

After proving that planetary elliptical orbits can be stable, he said, "Je n'avais pas besoin de cette hypothèse-là" or "I had no need of that hypothesis" when asked by Napoleon why he did not invoke God to explain solar-system stability, as Newton had thought necessary because of chaotic conditions (which are there but just small enough).

**Epistemology**

Given physical laws and particle motions and positions, people can predict everything in the future.

**Metaphysics**

Solar system formed from spinning gas cloud {nebular hypothesis}. Gravity and motion correct planetary-orbit perturbations, rather than causing chaos.

**George Green [Green, George]**

mathematician  
 England  
 1827

Essay on the Application of Mathematical Analysis to the Theories of Electricity and Magnetism [1827]

He lived 1793 to 1841, invented Green's theorem, and studied double integrals, line integrals, and curvilinear integrals.

**Joseph Liouville [Liouville, Joseph]**

mathematician  
 Paris, France  
 1829 to 1851

Journal of Pure and Applied Mathematics [1836]

He lived 1809 to 1882 and invented Sturm-Liouville theory [1829 and 1837] and transcendental numbers [1851]. Phase-space region volume is constant for Hamiltonian equation {Liouville's theorem, Liouville}, but volumes spread into larger space, leaving empty spaces.

**Gabriel Lamé [Lamé, Gabriel]**

mathematician  
 Paris, France  
 1840 to 1859

Lessons on curvilinear coordinates and their diverse applications [1859]

He lived 1795 to 1870 and studied curvilinear coordinates [1840] and invented Lamé's differential equation.

**George Gabriel Stokes [Stokes, George Gabriel]**

mathematician  
 Ireland  
 1845 to 1849

Dynamical Theory of Diffraction [1849]

He lived 1819 to 1903 and invented Stokes theorem [1845], fluid-dynamics Navier-Stokes equations, and Stokes lines. Navier-Stokes equations extend Newton's second dynamics law and linear constitutive stress relation.

**Paul Du Bois-Reymond [Du Bois-Reymond, Paul]**

mathematician  
 France  
 1877

On the paradoxes of the infinitary calculus [1877]

He lived 1831 to 1889 and classified partial differential equations.

**Thomas Stieltjes [Stieltjes, Thomas]**

mathematician

Netherlands

1894

Researches on continuous fractions [1894]

He lived 1856 to 1894 and invented Stieltjes integral.

**Richard V. Southwell [Southwell, Richard V.]**

mathematician

England

1940 to 1952

Relaxation Methods in Engineering Science [1940 and 1952]

He lived 1888 to 1970 and solved differential equations by substituting algebraic equations {relaxation method, Southwell}.

**Abraham Robinson [Robinson, Abraham]**

mathematician

Germany/USA

1951 to 1966

On the Metamathematics of Algebra [1951]; Non-Standard Analysis [1961 to 1966]

He lived 1918 to 1974 and developed the idea of infinitesimals as greater than zero but smaller than all positive numbers {nonstandard analysis, Robinson}. He described infinitesimal neighborhoods of points infinitely close to a point {compactness theorem, Robinson}.

**MATH>Mathematics>History>Complexity Theory**

**Andrei N. Kolmogorov [Kolmogorov, Andrei N.]**

mathematician

Russia

1933 to 1965

Foundations of Probability [1933]; General theory of dynamical systems and classical mechanics [1954]

He lived 1903 to 1987, invented Kolmogorov probability, and developed measure theory [1965]. System-complexity measures {algorithmic complexity, Kolmogorov} {Kolmogorov complexity, Kolmogorov} {algorithmic information content, Kolmogorov} can be number of bits for smallest program that can run on universal Turing machines and produce same output. In turbulence, low frequencies transfer energy to higher frequencies throughout fluid.

**Mary Cartwright [Cartwright, Mary]/John Littlewood [Littlewood, John]**

mathematician

England

1946

She lived 1900 to 1998. For non-linear radio amplifiers, equations {Van der Pol equation} can calculate output for sine-wave input. At higher amplifier gains, output period doubles input period and then becomes non-periodic. Van-der-Pol-equation solutions were early chaos-theory ideas.

**Lev D. Landau [Landau, Lev D.]**

physicist

Russia

1959

Course of Theoretical Physics [1959: with Evgenii M. Lifshitz]

He lived 1908 to 1968. He proposed neutron stars [1932], and J. Robert Oppenheimer and G. M. Volkov found mass limit {Landau-Oppenheimer-Volkov limit, Landau} for making black holes instead of neutron stars, 2.5 times Sun mass.

Turbulence begins when new frequencies appear in fluid at overlapping velocities and masses. Turbulent motions include oscillatory, skewed varicose, cross-roll, knot, and zigzag. Turbulence is like white noise, with all frequencies.

**Edward Lorenz [Lorenz, Edward]**

meteorologist/computer scientist

USA

1963

Deterministic Nonperiodic Flow [1963]

He lived 1917 to ? and studied complex systems. He invented non-periodic weather-system computer models that were sensitive to initial conditions {butterfly effect, Lorenz}.

He studied fluid convections with circular motions {Rayleigh-Bénard convection, Lorenz}. Equations are  $dx / dt = 10 * (y - x)$ ,  $dy / dt = x * z + 28 * x - y$ , and  $dz / dt = x * y - (8/3) * z$ .

Paths through phase space never cross. Attractor can move to another surface when it moves to another phase-space region, so surfaces do not intersect.

Complex non-linear systems can have different final states that are not interchangeable {intransitive system}. Systems can be almost intransitive and can flip spontaneously from one state to another.

**Alexander N. Sarkovskii [Sarkovskii, Alexander N.]**

mathematician

Russia

1964

Coexistence of Cycles of a Continuous Map of a Line into Itself [1964]

One-dimensional objects with cycle of period three have all periods.

**Stephen Smale [Smale, Stephen]**

mathematician

USA

1967

Differentiable Dynamic Systems [1967]

He lived 1930 to ? and studied non-linear oscillators that had stable, non-repeating, periodic patterns. He studied topology in five or higher dimensions and Poincaré conjecture. He invented topological phase-space transformations {Smale's horseshoe}, in which space stretches, shrinks, and folds multiple times in any dimension. Transformations are sensitive to initial conditions.

**René Thom [Thom, René]**

mathematician

France

1968 to 1972

Dynamic theory of morphogenesis [1968]; Structural Stability and Morphogenesis [1972]

He lived 1923 to 2002 and studied catastrophe theory.

**David Ruelle [Ruelle, David]**

mathematician

Belgium

1971

On the Nature of Turbulence [1971: with Floris Takens]

He used three independent motions to describe turbulence. However, this was wrong. Phase-space centers can be not equilibria or periodic loops but infinitely long lines in confined space {strange attractor, Ruelle}. Strange attractors are stable, can have few dimensions, and are periodic but not exactly periodic.

**Mitchell Feigenbaum [Feigenbaum, Mitchell]**

physicist

USA

1973

He studied feedback systems and devised how to calculate order in one-dimensional-system chaos [1973], using quantum-field-theory renormalization group, stochastic processes, and fractals to remove infinities. Using  $y = r * (x - x^2)$  and  $x(t) = r * \sin(\pi * x(t - 1))$ , doubling oscillation period converges geometrically and so scales with constant

ratio = 4.6692016090, to predict all doubling values. Functions are recursive {self-referential} and so introduce higher frequencies that indicate turbulence.

**Harry Swinney [Swinney, Harry]**

physicist  
USA  
1973

He studied conductivity [1973], with Jerry Gollub. He studied phase transitions. Rotating one cylinder inside another causes intervening liquid to flow {Couette-Taylor flow, Swinney}. First, flow streamlines. At faster speed, fluid cylinder separates into layers along cylinder axis, so fluid goes up and down cylinder. At higher frequency, flow is chaotic, with no defined frequencies. Vapor at critical point gives off white glow {opalescence, vapor}.

**James Yorke [Yorke, James]**

mathematician  
USA  
1975

Period Three Implies Chaos [1975: with Tien-Yien Li]

He analyzed work of Robert May. In one-dimensional systems, regular cycle of period three implies regular cycles of other periods, as well as chaotic behavior.

**Robert May [May, Robert]**

biologist  
USA  
1976

Simple Mathematical Models with Very Complicated Dynamics [1976]

Assign initial number to logistic difference equation. Low rate values make number go to zero. Medium values make number go higher steady-state numbers. After high initial value, system oscillates between two values. After even higher initial value, system oscillates among four values. After even higher initial values, system oscillates among 8, 16, 32, and so on, values, with smaller differences between rates, until chaos starts {point of accumulation} {accumulation point, complexity}. After that point, oscillations are among all values. However, at higher points, oscillations are among 3 or 7 values, then oscillations are among 6, 9, 12, 14, 21, 28, and so on, values, then chaos returns again.

**Stanislaus M. Ulam [Ulam, Stanislaus M.]**

mathematician  
Poland/USA  
1976

Adventures of a Mathematician [1976]

He lived 1909 to 1984 and studied chaos in vibrating strings {Fermi-Pasta-Ulam theorem}.

**Michel Hénon [Hénon, Michel]**

astronomer  
France  
1976 to 2002

He studied stretching, compressing, and folding phase space to get self-similarity {Hénon attractor} [1976]:  $x(t) = y(t - 1) + 1 - 1.4 * (x(t - 1))^2$  and  $y(t) = 0.3 * x(t - 1)$ . He predicted that globular clusters have center that experienced gravitational collapse {gravothermal collapse} [2002].

**Alexander Woodcock [Woodcock, Alexander]/Monte Davis [Davis, Monte]**

mathematician  
USA  
1978

Catastrophe Theory [1978]

They studied catastrophe theory.

**Stuart Kauffman [Kauffman, Stuart]**

mathematician

USA

1993 to 1995

Origins of Order [1993]; At Home in the Universe [1995]

He studied random graphs and Boolean networks to try to find complex-system, chaos, and self-organization laws. Most algorithms are their shortest descriptions {incompressibility}. Element physical interactions can order systems {self-organization, Kauffman} [Kauffman, 1995].

### **Brian Goodwin [Goodwin, Brian]**

mathematician

USA

1994

How the Leopard Changed Its Spots: The Evolution of Complexity [1994]

Self-organizing systems follow physical laws and describe living-system energy flows.

### **Alwyn Scott [Scott, Alwyn]**

mathematician

USA

1995 to 2005

Stairway to the Mind [1995]; Neuroscience: A Mathematical Primer [2002]; Nonlinear Science: Emergence and Dynamics of Coherent Structures [2003: 2nd edition]; Encyclopedia of Nonlinear Science [2005: editor]

Brain has hierarchical structure and new properties can arise at highest levels.

### **Albert Libchaber [Libchaber, Albert]**

physicist

France

1996

He used a liquid-helium box to study turbulence onset and found that it had period doubling, as in other complex non-linear systems [1996]. First, system reaches steady state as cylinders roll, then convection rolls become toruses, then those bifurcate, making 1, 2, 4, 8, 16, and so on, rolls as convection coil goes faster, and turbulence increases.

### **Leo Kadanoff [Kadanoff, Leo]**

physicist

USA

1999

Phase transitions and critical points can be hierarchies of phase regions that affect neighbors {phase scaling} [1999].

## **MATH>Mathematics>History>Computer Science**

### **Ada Lovelace [Lovelace, Ada] or Augusta Byron [Byron, Augusta]**

mathematician

London, England

1843

Notes on the Analytical Engine [1843: notes about general purpose computers added to her translation of French memoir on Babbage's Analytical Engine]

She lived 1815 to 1852. Calculating machines cannot be creative, but only do what program indicates {Lady Lovelace's objection}.

### **Alan Mathison Turing [Turing, Alan Mathison]**

mathematician

Britain

1937 to 1950

On Computable Numbers with an Application to the Entscheidungs Problem [1937]; Computing Machinery and Intelligence [1950]

He lived 1912 to 1954 and developed Turing test for intelligence. He developed a code-breaking machine {electronic cryptanalytic machine}, which was the first programmed computer {Colossus computer}. Fixed definite

processes {algorithm, Turing} {recursive procedure, Turing} or trial-and-error procedure {heuristic procedure} can solve mathematical problems. Turing machines programmed to perform procedures can solve problems. Universal Turing machines can define all possible operations and solve general problems. Algorithms and heuristics cannot solve some mathematical problems, so machines cannot solve them [Turing, 1950].

**Abraham Wald [Wald, Abraham]**

mathematician

Hungary/USA

1939 to 1947

Sequential Analysis [1947]

He lived 1902 to 1950, studied statistical decision problem, and used minimax [1939].

**Norbert Wiener [Wiener, Norbert]**

mathematician

USA

1942 to 1958

Cybernetics or Control and Communication in the Animal and the Machine [1942 and 1947]; Human Use of Human Beings [1950]; Nonlinear Problems in Random Theory [1958]

He lived 1894 to 1964 and studied non-linear problems. He developed animal and machine control and communication theory {cybernetics, Wiener} and feedback-using self-regulating system theory. He helped develop artificial limbs. He studied automata {logical net}, information theory, and principles involved in communication between sources and sinks.

**Epistemology**

Information has encoding, transmission, and decoding. A possibly noisy channel transmits information. Channel has information capacity. The same information can use different codes, one of which can be optimum.

**John von Neumann [Neumann, John von]**

mathematician

Germany/USA

1944 to 1958

Theory of Games and Economic Behavior [1944 and 1953: with Oskar Morgenstern]; Probabilistic Logics [1952]; Mathematical Foundations of Quantum Mechanics [1932]; Computer and the Brain [1958]

He lived 1903 to 1957.

In logic, he studied empirical logic, logic with uncertainty, error, and logical-net errors and helped develop quantum logic, with Birkhoff and Mackey.

In computing, he studied linear programming and electronic digital-computer theory and developed first digital computer [1946], called ENIAC. Multiple connections between elements allow system to operate, even if some units fail {multiplexing, Neumann}. Multiple lines can provide multiplexing.

In biology, he studied finite automata as central-nervous-system models.

In geometry, he showed how to use general eigenvalue theory for axiomatic Hilbert spaces and operators.

In game theory, he studied zero-sum games, strategy, Colonel Blotto game, minimax theorem, utility function, prisoner's dilemma, competition, and cooperation. Game theory involves decision-making when conditions are uncertain.

Set theory does not allow sets {paradoxical set, Neumann} to be their own elements {Foundation axiom, Neumann}.

**Richard W. Hamming [Hamming, Richard W.]**

mathematician

USA

1950 to 1960

Coding and Information Theory [1960]

He lived 1915 to 1998 and invented Hamming code [1950] to detect computer-coding errors.

**Edward Fredkin [Fredkin, Edward]**

mathematician

USA

1956 to 2005

He lived 1934 to ? and invented reversible-computing gates {Fredkin gate} {Conservative Logic Gate}.  
Mathematical models can reversibly transform into computational models {Fredkin transforms}. Universe computes using discrete and finite quantities {digital mechanics}. The more two alternatives are similar, the harder it is to choose and the less the choice matters {Fredkin paradox, Fredkin}.

**Hao Wang [Wang, Hao]**

mathematician

China

1959 to 1986

Beyond Analytic Philosophy [1986]

He lived 1921 to ?, invented computer programs to prove first-order theorems [1959], and invented infinite series of types. Mathematics is intuitive.

**Seymour Papert [Papert, Seymour]**

mathematician

South Africa/USA

1991

Constructionism: research reports and essays 1985-1990 [1991: with I. Harel]

He lived 1928 to ?, studied learning theories {constructionism}, and invented the Logo computer language.

**Joseph Weizenbaum [Weizenbaum, Joseph]**

mathematician

USA

1960

Computer Power and Human Reason [1960]

He lived 1923 to ? and wrote ELIZA program to imitate psychologist querying patient.

**Scott E. Fahlman [Fahlman, Scott E.]**

mathematician

USA

1979

NETL, A System for Representing and Using Real World Knowledge [1979]

He studied neural networks.

**Patrick Henry Winston [Winston, Patrick Henry]**

psychologist

USA

1979

Artificial Intelligence, an MIT Perspective [1979: editor with Richard Henry Brown]

He studied AI.

**Avron Barr [Barr, Avron]/Edward A. Feigenbaum [Feigenbaum, Edward A.]**

mathematician

USA

1981

Handbook of Artificial Intelligence [1981]

They studied AI.

**Gregory E. Hinton [Hinton, Gregory E.]**

mathematician

USA

1981 to 1992

Parallel Models of Associative Memory [1981: editor with John A. Anderson]; How Neural Networks Learn from Experience [1992]

He invented backpropagation learning algorithms.

**Valentin Braitenberg [Braitenberg, Valentin]**

mathematician  
USA  
1984  
Vehicles [1984]  
He invented robots.

**W. Daniel Hillis [Hillis, W. Daniel]**

mathematician  
USA  
1985  
Connection Machine [1985]  
He studied neural networks.

**David Deutsch [Deutsch, David]**

mathematician  
England  
1985 to 1993  
He studied quantum computation [1985].

**David Rumelhart [Rumelhart, David]/James McClelland [McClelland, James]**

mathematician  
USA  
1986  
Parallel Distributed Processing [1986]  
They studied neural networks, with Gregory E. Hinton and R. J. Williams.

**Ivars Ekeland [Ekeland, Ivars]**

mathematician  
USA  
1988  
Mathematics and the Unexpected [1988]  
He studied computer memory.

**Pentti Kanerva [Kanerva, Pentti]**

mathematician  
USA  
1988  
Sparse Distributed Memory [1988]  
He studied computer memory.

**Jeffrey Elman [Elman, Jeffrey]**

computer scientist  
USA  
1990  
Finding Structure in Time [1990]  
To hidden layer, he added units {context layer, Elman} that received a hidden-layer copy and then added back to hidden layer {simple recurrent network, Elman}.

**Terrence J. Sejnowski [Sejnowski, Terrence J.]**

mathematician  
USA  
1992  
He studied shape from shading in neural networks [1992], with Sidney Lehky.

**Stephen Wolfram [Wolfram, Stephen]**

mathematician

USA

1994 to 2002

Cellular Automata and Complexity: Collected Papers [1994]; New Kind of Science [2002]

He invented Mathematica software.

Science does not need laws expressed as mathematical equations. Simple non-linear rules operating on simple units can generate all pattern types and describe all phenomena. Because they can be equivalent to any algorithm, cellular automata can describe all complex processes. Physical systems satisfying differential equations can be cellular automata, by substituting finite differences and discrete variables for differential equations. His Rule 30 seems to create unpredictable pattern, rather than expected recursiveness.

[www.stephenwolfram.com/publications/articles/date.html](http://www.stephenwolfram.com/publications/articles/date.html).

### **Benjamin W. Schumacher [Schumacher, Benjamin W.]**

mathematician

USA

1996 to 1998

Quantum data processing and error correction [1996]

Information is only in physical media, which store bits or qubits {information science, Schumacher}. Physical medium can transform and/or transfer information to process information. Output from processing must be verifiable or complete task.

### **Scott Kirkpatrick [Kirkpatrick, Scott]**

mathematician

USA

2001

traveling-salesman problem [2001]

Salesmen want to travel shortest distance among cities, with no path duplication. What is the shortest path {traveling-salesman problem, Kirkpatrick} [2001]? Traveling-salesman problems are NP-complete. Number of possible paths is factorial of number of cities, divided by two, because trips can be in either direction. Tours are vertexes of N-dimensional polygons. Tours that differ by one city are near each other in N-dimensional space. Simulated annealing can find shorter paths but allow longer paths, to avoid local minima. Techniques can find good paths but not necessarily the best.

## **MATH>Mathematics>History>Game Theory**

### **Oskar Morgenstern [Morgenstern, Oskar]**

mathematician

USA

1944

Theory of Games and Economic Behavior [1944: with John von Neumann]

He lived 1902 to 1977 and studied game theory, competition, and cooperation.

### **John F. Nash, Jr. [Nash, John F. Jr.]**

mathematician

USA

1950 to 1951

Equilibrium points in n-person games [1950]; Bargaining problem [1950]; Non-cooperative games [1951]; Two-Person Cooperative Games [1953]

He lived 1928 to ? and invented Nash equilibrium [1950], Nash bargaining solution [1950], and Nash programme [1951].

## **MATH>Mathematics>History>Geometry**

### **geometric patterns**

mathematician

France

-25000  
geometric patterns  
Cavemen painted designs on cave walls.

### **Moscow Papyrus**

mathematician  
Thebes, Egypt  
-1900  
Moscow Papyrus or Golenischev Papyrus [-1900]  
Papyrus describes Egyptian geometry.

### **Pythagorean theorem proved**

mathematician  
Babylonia  
-1850 to -1750  
Pythagorean theorem  
Babylonian mathematicians used Pythagorean theorem to find distances.

### **Manava**

Vedic priest/craftsman  
India  
-720  
Discourses on Altar Construction [-720: about altar construction]  
He lived -750 to -690 and constructed circles from rectangles and squares from circles.

### **Sophists**

mathematics school  
Greece  
-450 to -300  
Sophists invented geometric proofs and studied circles as many-sided regular polygons. They tried to square circle, trisect angle, and double cube using only straightedge and compass.

### **Hippocrates of Chios**

mathematician  
Athens, Greece  
-450 to -410  
He lived -470 to -410 and wrote first geometry text, first calculated curved area using rectilinear figures {quadrature, Hippocrates}, and first proved theorems using earlier theorems {pyramiding theorems}. He invented method of proving something by disproving its opposite {indirect proof, Hippocrates}.

### **Eudoxus of Cnidus**

astronomer/mathematician  
Cnidus, Greece  
-380 to -355  
He lived -408 to -355. He studied limits, used infinite polygons to find curved-figure areas and volumes {exhaustion method, Eudoxus}, and developed explicit axioms.

Proportion is magnitude or length. He showed how to prove that two different integer ratios, which make real numbers, are equal or not equal. Proportions are magnitude or length ratios. To compare ratios, find integer pairs such that product of first integer and numerators and product of second integer and denominators makes numerators greater than denominators. If successful, first ratio is greater than second, because new ratio, first/second, is less than first ratio and greater than second ratio. If unsuccessful, find integer pairs such that product of first integer and numerators and product of second integer and denominators makes numerators less than denominators. If successful, first ratio is less than second, because new ratio, first/second, is greater than first ratio and less than second ratio. If not successful, ratios are equal. You can thus approach any real number and so can work with irrational-number square roots of positive integers.

Planetary orbits are nested spheres. He measured year length.

## **Euclid**

mathematician

Alexandria, Egypt

-300 to -280

Elements [-300 to -280]

He lived -325 to -265 developed Euclid's theorem and Euclid's algorithm. He studied perpendicular, parallel, superposition, arc, and prime numbers. He used exhaustion method, rather than infinitesimals, to study curves. He systematized plane geometry, number proportions and ratios, prime numbers, and solid geometry.

Book 1 is about congruence, parallel lines, Pythagorean theorem, simple constructions, constructions with equal areas, and parallelograms {rectilinear figure, Euclid}. Sum of two triangle-side lengths is greater than or equal to third-side length. Book 2 is about geometric algebra, using areas and volumes to find products and quadratic equations, and adding line segments to add. Book 3 is about circles, chords, tangents, secants, central angles, and inscribed angles. Book 4 is about figures inscribed in, or circumscribed around, circles. Book 5 is about proportion by magnitudes, commensurable magnitudes, and incommensurable magnitudes. Book 6 is about similar figures, using proportions. Book 7 is about number theory, Euclidean algorithm, and numbers as line segments. Book 8 is about geometric progressions. Book 9 is about square and cubic numbers, plane and solid numbers, geometric progressions, and the theorem that number of primes is infinite. Book 10 classifies incommensurable magnitudes. Book 11 is about convex solids and generation of solids. Book 12 is about curved-surface areas and volumes, using exhaustion method and indirect proof. Book 13 is about regular polyhedrons in spheres and regular polygons in circles.

## **Eratosthenes**

mathematician

Egypt

-250 to -200

He lived -276 to -194 and found circumference of Earth.

## **Apollonius of Perga or Appolonios or Great Geometer**

mathematician/philosopher

Greece

-230 to -200

Conics or Conic Sections [-230 to -200]

He lived -262 to -185 and was Neo-Pythagorean and mystic. He invented a systematic theory of parabolas, ellipses, and hyperbolas, based on eccentricity, directrix, and focus. He studied right circular cones, oblique circular cones, hyperbolas, parabolas, ellipses, conjugate diameters, tangents, asymptotes, foci, conic intersections, maximum and minimum conic lengths, conic normals, similar and congruent conics, and conic segments. Two conic tangents meet at poles, and sides are polars. Given three points, lines, or circles, construct a circle tangent to or including the points, lines, or circles {Apollonian problem}.

## **Ethics**

Simple life is best.

## **Mind**

Mind and body are separate realities.

## **Ptolemy**

mathematician

Alexandria, Egypt

150

Almagest or Great Book [150]

He lived 87 to 150, invented maps with longitude and latitude, discovered Ptolemy's theorem, and invented epicycles to describe planetary motions.

## **Pappus of Alexandria**

mathematician

Alexandria, Egypt

340

Mathematical Collection or The Collection [340]

He lived 260 to 350 and proved Pappus' theorem {Guldinus theorem}.

**Abul Wafa Muhammad al-Buzjani [al-Buzjani, Abul Wafa Muhammad] or al-Buzjani**

mathematician

Baghdad, Iraq

970 to 980

Book on What Is Necessary from the Science of Arithmetic for Scribes and Businessmen [970 to 980]; Book on What Is Necessary from Geometric Constructions for the Artisan [970 to 980]

He lived 940 to 997, used secant and cosecant, and constructed using straightedges and circles.

**Nasir-Eddin or Nasireddin or Nasir Tusi [Tusi, Nasir] or Abu Jafar Muhammad Ibn Muhammad Ibn al-Hasan Nasir al-Din al-Tusi [al-Tusi, Abu Jafar Muhammad Ibn Muhammad Ibn al-Hasan Nasir al-Din] or Nasir al-Din al-Tusi [al-Tusi, Nasir al-Din]**

mathematician

Alamut, Persia

1272

Ilkhanic Tables [1272]

He lived 1201 to 1274 and used tangent and secant. He invented devices to resolve linear motion into sum of two circular motions {Tusi-couple, Nasir-Eddin}.

**François Vieta [Vieta, François]**

mathematician

Paris, France

1579

Mathematical Canon [1579]

He lived 1540 to 1603 and invented sine law, cosine law, and Napier's rule.

**Simon Stevin [Stevin, Simon]**

mathematician

Bruges, Belgium

1585

Art of Tenths [1585]

He lived 1548 to 1620 and used decimal fractions and force parallelograms.

**Bonaventura Cavalieri [Cavalieri, Bonaventura]**

mathematician

Bologna, Italy

1629 to 1647

Geometry of Indivisibles [1635]; Geometric Exercises [1647]

He lived 1598 to 1647, invented Cavalieri's theorem, and studied indivisibles method [1629].

**Girard Desargues [Desargues, Girard]**

mathematician

France

1639

Rough draft for an essay on the results of taking plane sections of a cone [1639]

He lived 1591 to 1661, invented Desargue's theorem, and studied projective geometry, involution, harmonic point sets, and poles and polar theory.

**Alexis-Claude Clairaut [Clairaut, Alexis-Claude]**

mathematician

Paris, France

1731 to 1752

Theory of the Shape of the Earth [1743]; Theory of the Moon [1752]

He lived 1713 to 1765, studied space curves [1731], invented Clairaut's equation, and determined Earth's shape.

**Maria Gaetana Agnesi [Agnesi, Maria Gaetana]**

mathematician

Bologna, Italy

1748

Analytic Institutions for Use by Italian Youth [1748 to 1749]

She lived 1718 to 1799 and published discussion of cubic witch of Agnesi curve [1948].

**Gaspard Monge [Monge, Gaspard]**

mathematician

Paris, France

1768 to 1800

Descriptive Geometry [1800]

He lived 1746 to 1818, studied developable surfaces, rediscovered projective geometry [1768], and was the "father of descriptive geometry".

**Jean-Victor Poncelet [Poncelet, Jean-Victor]**

mathematician

Paris, France

1822

Treatise on the projective properties of figures [1822]

He lived 1788 to 1867, rediscovered projective geometry, and studied affine geometry, differential geometry, and harmonic point sets.

**Janos Bolyai [Bolyai, Janos]**

mathematician

Hungary

1823 to 1833

Appendix to Tentamen [1833: of Farkas Bolyai or Wolfgang Bolyai]

He lived 1802 to 1860 and used substitute parallel axiom [1823], applied to intersecting and non-intersecting lines, to make non-Euclidean geometry [1833].

**Julius Plucker [Plucker, Julius]**

mathematician

Germany

1828 to 1835

Developments in Analytic Geometry [1828 to 1831: two volumes]; System of Analytic Geometry [1835]

He lived 1801 to 1868 and studied trilinear coordinates and line coordinates.

**Joseph Plateau [Plateau, Joseph]**

mathematician

Paris, France

1836 to 1883

Experimental and Theoretical Statics of Liquids under only Molecular Forces [1873]; stroboscope [1836]

He lived 1801 to 1883 and invented Plateau's problem.

**Karl von Staudt [Staudt, Karl von]**

mathematician

Nuremberg, Germany

1847

Geometry of Position [1847]

He lived 1798 to 1867 and analyzed projective geometry without metric and without congruence.

**Georg Bernhard Riemann [Riemann, Georg Bernhard]**

mathematician

Germany

1854 to 1859

On the hypotheses that lie at the foundations of geometry [1854]; Theory of Abelian functions [1857]; On the number of primes less than given magnitude [1859]

He lived 1826 to 1866. He studied non-Euclidean geometry, differential geometry, complex functions, multiple-valued functions, mapping, prime-number theorems, analytic number theory, and singularities. He invented Riemann surfaces, Riemann-Darboux integral, Riemann zeta function, Riemann mapping theorem, and Riemann hypothesis. Riemann integrals are sums over infinity of step functions. All closed line segments have the same number of points. All points, in plane touching Riemann sphere at South Pole, map to sphere points, with points at infinity mapping to North Pole. Compact-plane points can thus map to limited, closed, and bounded surfaces.

**Edwin A. Abbott [Abbott, Edwin A.] or A. Square [Square, A.]**

mathematician

England

1926

Flatland A Romance of Many Dimensions [1926]

He lived 1838 to 1926.

**Benoit Mandelbrot [Mandelbrot, Benoit]**

mathematician

Poland/France/USA

1977

Fractals: Form, Chance and Dimension; Fractal Geometry of Nature [1977]

He lived 1924 to ? and ascribed fluctuations to discontinuous effects and to trends. He studied fractals, self-symmetry,  $1/f$  noise, and  $1/f$  squared noise. Fractal curves have non-integral dimensions.  $1/f$  noise is like Cantor sets. In continuous intervals, continually removing inner third of each remaining continuous interval still leaves infinitely many points, and total empty distance is interval length {Cantor set, Mandelbrot}. Cantor sets are the same at all scales.

**Ian Stewart [Stewart, Ian]**

mathematician

England

1989 to 2001

Does God Play Dice? [1989]; Flatterland [2001]

He lived 1945 to ?.

**MATH>Mathematics>History>Group Theory**

**Evariste Galois [Galois, Evariste]**

mathematician

France

1830 to 1832

He lived 1811 to 1832 and studied group, field, solvability, and factoring representation theory [1830 to 1832].

**Sophus Lie [Lie, Sophus]**

mathematician

Germany

1874

Theory of Integrability Factors [1874]

He lived 1842 to 1899 and studied transformation groups and finite continuous groups {Lie group, Lie}.

**Walther von Dyck [Dyck, Walther von]**

mathematician

Netherlands

1901

He lived 1856 to 1934 and combined equation group theory, group number theory, and infinite transformation groups {abstract group theory} [1901].

**Emmy Noether [Noether, Emmy] or Amalie Noether [Noether, Amalie]**

mathematician  
Göttingen, Germany/USA  
1915 to 1921  
Ideal Theory in Ring-Fields [1921]  
She lived 1882 to 1935, studied invariance, and showed that symmetries relate to conservation laws [1915] {Noether's theorem, Noether}. She also studied rings [1921].

#### **MATH>Mathematics>History>Information Theory**

##### **Claude Shannon [Shannon, Claude]**

mathematician  
USA  
1948  
Mathematical Theory of Communication [1948]  
He lived 1916 to 2001 and founded information theory and studied transition probabilities.

#### **MATH>Mathematics>History>Invention**

##### **Archimedes**

physicist/mathematician/inventor  
Syracuse, Sicily  
-264 to -212  
Archimedes' screw  
He lived -287 to -212 and invented Archimedes' theorem, Archimedes spiral, Archimedes axiom, and Archimedes real-number property. He used exhaustion method to find pi and sphere and conic areas and volumes. He used completeness axiom. He found Archimedes buoyancy law {Archimedes' principle, Archimedes} {Archimedes principle, Archimedes}. He put a screw {Archimedes' screw} {Archimedes screw} inside a cylinder, to lift water.

##### **Hero of Alexandria**

mathematician/physicist/inventor  
Alexandria, Egypt  
60 to 62  
Pneumatics [60]; Automata [62]; Mechanics [60 to 70]; Metrics [60 to 70]; Sighting Tube [60 to 70]  
He lived 10 to 70, invented Hero's formula, and studied geodesy, mechanics, and pneumatics. He maintained constant water-clock water supply, using float and needle valve, as in carburetors. He invented steam engine {aeolipile} [62].

##### **Blaise Pascal [Pascal, Blaise]**

mathematician/philosopher/inventor  
France  
1642 to 1670  
Thoughts [1670]; Provincial Letters; calculating machine [1642]  
He lived 1623 to 1662, was Cartesian and Jansenist, and invented first metal-tooth wheeled calculating machine [1642]. He invented hydraulic press to multiply force, syringe, Pascal's principle, Pascal's theorem, Pascal's triangle, mathematical induction, fundamental enumeration principle, binomial theorem, large-numbers law, and conditional-probability law.  
At mechanical equilibrium, with only gravity acting, liquid has hydrostatic pressure {Pascal's law}.

##### **Epistemology**

People can neither reject reasoning nor say there is only reasoning. Reason cannot deal with ultimate metaphysical problems. Faith is necessary complement to reason. Expected value of believing in God is more than value of non-belief {Pascal's wager}.

##### **Metaphysics**

God exists because man is helpless without God.

##### **John G. Kemeny [Kemeny, John G.]**

mathematician/inventor

Hungary/USA

1964

BASIC [1964]

He lived 1926 to 1992 and developed a FORTRAN-like programming language {BASIC programming language}.

## **MATH>Mathematics>History>Logic**

### **George Boole [Boole, George]**

mathematician

England

1847 to 1854

Mathematical Analysis of Logic [1847]; Investigation of the Laws of Thought, on which are founded the Mathematical Theories of Logic and Probabilities [1854]

He lived 1815 to 1864 and studied symbolic logic and logic of classes or extensional logic. Arithmetic and algebras have axioms and theorems allowing independent term or variable meanings. Axioms and theorems can be statements, sets, classes, events, or durations. Syllogisms can use arithmetic notation, and algorithm can prove them {Boolean algebra, Boole}. Boolean algebra has sets, union operation, intersection operation, complement operation, zero element, and unit element. Arithmetic axioms hold for elements and operations.

#### **Epistemology**

Mind has ability to conceive class, designate individual class members by common name, perform other logical tasks, and think logically {laws of thought, Boole}. Thought laws are innate and inherited.

### **Joseph Bertrand [Bertrand, Joseph]**

mathematician

Paris, France

1889

Calculation of Probabilities [1889]

He lived 1822 to 1900. Because circle chords can have varying angles to tangents, for example perpendicular to radius and parallel to tangent, different ways of selecting chords lead to different probabilities that chord is less than inscribed-equilateral-triangle side {Bertrand's paradox}.

### **Charles Dodgson [Dodgson, Charles] or Lewis Carroll [Carroll, Lewis]**

mathematician

England

1895

What the Tortoise Said to Achilles [1895]

He lived 1832 to 1898 and studied symbolic logic. Assuming inference rule is not the same as assuming conditional statement.

### **Jules Antoine Richard [Richard, Jules Antoine]**

philosopher

France

1905

He lived 1862 to 1956. Integers are describable in words with a finite number of letters. An integer exists that is the least integer not describable in 100 or less letters. However, that phrase has less than 100 letters {Richard's paradox} [1905].

### **Luitzen E. J. Brouwer [Brouwer, Luitzen E. J.]**

mathematician

Netherlands

1908 to 1924

Unreliability of the Logical Principles [1908]; Intuitionistic Reflections on Formalism [1927]

He lived 1881 to 1966, tried to define numbers, and helped develop quantum logic. He helped develop the idea that mathematics requires mental constructions for truth {intuitionism, Brouwer} [1924]. Unconstructed and non-existent things cannot be the basis for truth. Infinities cause excluded-middle-law contradiction, so mathematics cannot use this law.

**Alfred North Whitehead [Whitehead, Alfred North]**

mathematician/philosopher

Britain/USA

1910 to 1938

Principia Mathematica or Principles of Mathematics [1910 to 1913: with Russell]; Enquiry Concerning the Principles of Natural Science [1919]; Concept of Nature [1920]; Principle of Relativity with Applications to Physical Science [1922]; Science and the Modern World [1925]; Religion in the Making [1926]; Process and Reality [1929]; Adventures of Ideas [1933]; Modes of Thought [1938]

He lived 1861 to 1947 and was idealist. He studied logical analysis, axiomatized logic, and developed logicism. Events can relate {process, Whitehead}. Relations and events transform object properties. Objects are always changing properties or property values. Reality is about such changes {process philosophy, Whitehead}. Since no properties exist for significant times, processes and relations are more important than matter, time, and position. All things interconnect and continually adjust to environment {philosophy of organism, Whitehead}. Higher properties emerge from lower systems. God is always becoming, and this unifies universe. Qualities are not substances but are mind-activity results.

**Nicolai A. Vasiliev [Vasiliev, Nicolai A.]**

mathematician

Russia/Berlin, Germany

1911 to 1913

Imaginary (non-Aristotelian) Logic [1912: non-Aristotelian logic]; Logic and metalogic [1913]

He lived 1880 to 1940 and helped develop three-valued logic [1910 to 1913].

**Henry M. Sheffer [Sheffer, Henry M.]**

mathematician

USA

1913

Set of Five Independent Postulates for Boolean Algebras, with application to logical constants [1913]

He lived 1883 to 1964. Elements {Sheffer stroke element} can equal "Not AND" and fire if either, but not both, of two input elements fire. Sheffer-stroke-element combinations can make OR element, AND element, and NOT element. Using many Sheffer stroke elements creates devices whose output fires if and only if most inputs fire.

**Thoralf Skolem [Skolem, Thoralf]/Leopold Löwenheim [Löwenheim, Leopold]**

mathematician

Norway/Germany

1915 to 1920

Skolem lived 1887 to 1963. Löwenheim lived 1878 to 1957. If countable sets have formal models, domain is countable {Löwenheim-Skolem theorem}, as proved by Löwenheim [1915] and Skolem [1920]. However, real numbers are not countable {Skolem paradox}. Models {nonstandard model} can have elements that are not countable.

**Stanislaw Lesniewski [Lesniewski, Stanislaw]**

logician

Poland/Russia

1916

General Theory of Sets [1916]

He lived 1886 to 1939 and invented definition theory. He helped develop quantum logic, based on equivalence {protothetic logic, Lesniewski}, abstract quantifiers {ontology logic, Lesniewski}, and part and whole relations {mereology, Lesniewski}. Logic is not about real world, only about statements. Wholes are not just sets or sums of parts, because parts relate. Because living things can replace parts, modal or temporal logic can maintain integrated wholes by maintaining relations among replaced parts.

**Emil Post [Post, Emil]**

mathematician

USA

1920 to 1936

Introduction to a General Theory of Elementary Propositions [1920]

He lived 1897 to 1954. Symbol strings can substitute other symbol strings {Post grammar, Post} {Post machine} [1936], to make formal systems. Start with long symbol string and substitute, using symbol-string precedence rules.

Logic can be three-valued {many-valued logic}. Many-valued logic can use cyclic negation, so next truth-value negates previous one. Such systems include all finite-valued logics. Such logics can represent switching circuits with many inputs and/or outputs.

#### **Hans Reichenbach [Reichenbach, Hans]**

logician

Berlin, Germany

1928 to 1951

Philosophy of Space and Time [1928]; Rise of Scientific Philosophy [1951]

He lived 1891 to 1953, studied analytic philosophy, and helped develop quantum logic. Spaces and times are relative. Probability depends on frequency. Induction depends on frequency. The geometry people use for universe is just conventional, not real, because instruments can systematically alter from expectations.

#### **Clarence I. Lewis [Lewis, Clarence I.]**

logician

USA

1929 to 1959

Mind and the World Order [1929]; Symbolic Logic [1932: with Cooper H. Langford]; Analysis of Knowledge and Valuation [1946]

He lived 1883 to 1964, helped develop modal or relevance logic, developed implication requiring necessity {strict implication, Lewis}, and studied phenomenalism.

#### **Arend Heyting [Heyting, Arend]**

mathematician

England

1930

He lived 1898 to 1980 and helped develop quantum logic [1930].

#### **Stanislaw Jaskowski [Jaskowski, Stanislaw]**

mathematician

Poland

1934 to 1936

He lived 1906 to 1965 and invented natural deduction and worked with infinite-valued logic [1934 to 1936].

#### **Gerhard Gentzen [Gentzen, Gerhard]**

logician

Germany

1935

He lived 1909 to 1945. He developed formal first-order logic {natural deduction, Gentzen} [1935], which only assumes inference laws. One rule uses premises and operator to make compound statement {introduction rule, Gentzen}. Another rule uses compound statement and statement to make statement. Statements depend on simple and compound sequent statements. Sequent-calculus proofs can be truth-trees or truth-tables {cut elimination theorem, Gentzen}, which eliminate formulas. Natural deduction led to proof theory.

#### **Alfred Tarski [Tarski, Alfred]**

logician/mathematician

Poland/USA

1935 to 1983

Concept of Truth in Formalized Languages [1933]; On the Concept of Logical Consequence [1936]; Introduction to Logic and to the Methodology of Deductive Science [1937]; Semantic Conception of Truth [1944]; Undecidable Theories [1953]; Axiomatic Method: with special reference to geometry and physics [1957]; Equational logic and equational theories of algebra [1968]; Logic, Semantics and Metamathematics [1983]

He lived 1902 to 1983, founded modern logical theory, studied part and whole relations {mereology, Tarski}, helped develop quantum logic, and invented Banach-Tarski theorem.

Convention establishes basic-linguistic-element use and meaning {basic vocabulary}, which can construct complex term and sentence meanings {compositional semantics} {recursive semantics}.

Formal languages have consistent syntax, in which sentences form correctly or not. Formal language uses objects to replace language variables and predicates to replace language functions {interpretation, Tarski}. Truth is about interpretation {semantic theory of truth}. Determining truth requires defining what constitutes satisfying interpretation {satisfaction, Tarski}, which requires metalanguage {Tarski's theorem}. Formal languages have true interpretations {model, Tarski}. Premise sets can be models. If premise model is sentence model, sentences are premise-set consequences {theory of logical consequence} {logical consequence theory}.

For two sentence systems, sentences in one system can derive from sentences in other system {equipollence, Tarski}.

### **Kurt Grelling [Grelling, Kurt]**

philosopher  
Germany/England  
1936

Logical Paradoxes [1936]

He lived 1886 to 1941. Self-applicable can mean thing expresses property that it has. Self-applicable can mean expression applies to itself. If heterological means not-self-applicable, then heterological is both self-applicable and not-self-applicable {Grelling's paradox} {Weyl's paradox}.

### **Alonzo Church [Church, Alonzo]**

mathematician  
USA  
1941 to 1956

Calculus of Lambda-Conversion [1941]; Introduction to Mathematical Logic [1944 and 1956]

He lived 1903 to 1995, studied denotation, and helped develop quantum logic. Symbol strings can represent numbers and functions. Using functions on input function and data strings makes output function and data strings {lambda calculus, Church}. Lambda acts on variable or function, or variable and function combination, which is second-function dummy variable:  $\lambda(x(f(x))) = f$ ,  $\lambda(x(f(x)))(a) = f(a)$ ,  $\lambda(f(f(f(x)))) = \lambda(f(\lambda(x)(f(f(x)))))) = \lambda(f(x)(f(f(x))))$ . This expression is a function and precedes a value, which substitutes into function. In particular, after lambda, expressions can have variable zero times, function of variable one time, function of function of variable two times, and so on:  $0 = \lambda(f(x)(x))$ ,  $1 = \lambda(f(x)(f(x)))$ ,  $2 = \lambda(f(x)(f(f(x))))$ . Function of function equals lambda and function of function {abstraction, lambda calculus}:  $f(f(x)) = \lambda(x)(f(f(x)))$ . Really, symbols are functions. Lambda calculus represents recursion, iteration, and algorithm loops. Recursive functions can be equation sets. Recursive functions are computable {Church's theorem}. Functions are computable if they are recursive {Church's thesis, recursion}. Recursive functions can be lambda calculus. Lambda calculus is equivalent to Post grammar and Turing machine and so can express all algorithms. LISP computer language depends on lambda calculus.

### **Epistemology**

Formal systems can prove most theorems {effectively calculable} {computability}. Lambda calculus shows that it is impossible to prove some valid theorems in most formal systems, including arithmetic.

### **Stephen Cole Kleene [Kleene, Stephen Cole]**

mathematician  
USA  
1952

Introduction to Metamathematics [1952]

He lived 1909 to 1994, studied recursion theory and formal logic, and added subtraction to lambda calculus. At least one mathematical truth is true intuitionistically but not Platonically [Kleene, 1952].

### **Ruth C. Barcan [Barcan, Ruth C.] or Ruth C. Barcan Marcus [Barcan Marcus, Ruth C.]**

philosopher  
England  
1961 to 1993

Modalities and Intentional Languages [1961]; Modalities: Philosophical Essays [1993]

She lived 1921 to ? and studied modal logic. The possibility that something has an attribute implies that something exists that possibly has the attribute {Barcan formula}, assuming that possible worlds overlap.

**Lofti Zadeh [Zadeh, Lofti]**

mathematician  
Azerbaijan/USA  
1965  
fuzzy logic [1965]  
He lived 1921 to ? and invented fuzzy-set theory or fuzzy logic.

**George Spencer-Brown [Spencer-Brown, George]**

logician  
England  
1969  
Laws of Form [1969]  
He lived 1923 to ? and developed laws of form {calculus of indications, Spencer-Brown}, based on differences. Autopoietic theory references his work.

**Alan Ross Anderson [Anderson, Alan Ross]/Nuel D. Belnap, Jr. [Belnap, Jr., Nuel D.]**

philosopher  
USA  
1975 to 1992  
Entailment: The Logic of Relevance and Necessity [1975 and 1992]  
They helped develop relevance logic, modal logic, deontic logic, and logical connectives.

**Ray E. Jennings [Jennings, Ray E.]**

mathematician  
USA  
1986  
Punctuational Sources of the Truth-Functional "Or" [1986]  
Granting permission for two things can sound like permitting first or second, and so like exclusive OR, but is actually conjunction {confectionary fallacy, Jennings}. It is a deduction fallacy.

**MATH>Mathematics>History>Matrix**

**Arthur Cayley [Cayley, Arthur]**

mathematician  
England  
1855  
Determinants used before Matrices [1855]  
He lived 1821 to 1895 and studied matrix theory and invariant theory.

**MATH>Mathematics>History>Number Theory**

**numbers recorded**

mathematician  
France  
-30000  
recorded number  
Cavemen carved numbers on bones.

**sexagesimal number began**

mathematician  
Mesopotamia  
-3000  
sexagesimal number system

Sumerian sexagesimal number system was for finances, with positional notation but with no zero. They had ordinal numbers, cardinal numbers, odd and even numbers, addition, subtraction, and simple fractions. They also used lines, circles, and angles.

### **decimal number system**

number system

Egypt

-2900

decimal number system [-2900]

Egyptian mathematicians used hieroglyphs to represent 1, 10, 100, 1000, 10000, and 100000.

### **decimal number system**

number system

India/Pakistan

-2000

decimal number system [-2000]

Harappans decimal system was for weights and lengths.

### **multiplication table**

mathematician

Babylonia

-1800 to -1750

multiplication table

Babylonian mathematicians calculated multiplication tables for number squares, cubes, and square roots, using sexagesimal number system with positional notation.

### **decimal number system**

number system

China

-1360

decimal number system [-1360: decimal number system]

It had nine symbols for numerals 1 through 9, but zeroes were empty spaces.

### **counting board**

calculator

China

-1000

counting board [-1000]

Manipulating objects can add and subtract.

### **abacus**

calculator

Greece/Middle East/China

-1000 to -500

abacus [-1000 to -500: calculating frame with sliding beads]

The word abacus comes from Indo-European root for sand.

### **counting rods**

calculator

China

-540

counting rods [-540]

Manipulating red and black counting rods can add and subtract.

### **Puspendanta/Bhutabalin**

mathematician

India  
100 to 200  
Approaching the Parts [100 to 200: decimal number system]  
It has decimal logarithms.

### **Diophantus**

mathematician  
Greece  
250  
Arithmetic [250]  
He lived 200 to 284 and studied number theory, algebra symbols, and determinate and indeterminate equations.

### **Aryabhata or Aryabhata or Aryabhata**

mathematician  
Kusumapura (Patna), Bihar, India  
499  
Works of Aryabhata [499]; Principles of Aryabhata [499: about astronomical calculations]  
He lived 476 to 550, used positional notation, found circle chord lengths, and calculated sine tables.

### **Varahamihira or Varaha or Mihira**

mathematician  
Ujjain, Madhya Pradesh, India  
575  
Five Astronomical Canons [575]  
He lived 505 to 587 and used positional notation.

### **Bakhshali Manuscript**

mathematician  
Bakhshali, Pakistan  
600 to 700  
Bakhshali Manuscript [600 to 700]  
Manuscript is about arithmetic and algebra.

### **Brahmagupta**

mathematician  
Bhillamala (Bhinmal), Rajasthan, India  
628 to 665  
Opening of the Universe or Improved System of Brahma [628]; Time Sweetmeat [665]; zero; negative numbers  
He lived 598 to 668 and used decimal number system, negative numbers, and zero. He invented Brahmagupta's theorem.

### **Mahavira or Mahaviracharya**

mathematician  
India  
850  
Ganita Sara Sangraha or Compendium of the Essence of Mathematics [850]  
He lived 800 to 870 and used zero, positional notation, decimal system, and negative, irrational, and rational numbers.

### **Fibonacci or Leonardo of Pisa**

mathematician  
Pisa, Italy  
1202  
Book of Calculation [1202]  
He lived 1170 to 1250 and invented Fibonacci numbers and studied number theory.

**Simon Stevinus [Stevinus, Simon]**

mathematician

Netherlands

1585

Decimal Numbers [1585]

He lived 1548 to 1620. He invented decimal numbers and fractions, and so real numbers [1585].

**John Napier [Napier, John]**

mathematician

Edinburgh, Scotland

1594 to 1617

Description of the Wonderful Canon of Logarithms [1614]; Using Sticks for Calculation of Products [1617: about Napier's Rods or Napier's Bones]

He lived 1550 to 1617, used decimal point [1594], and studied logarithms.

**John Wallis [Wallis, John]**

mathematician

England

1655

He lived 1616 to 1703, studied cryptography, and invented expressions for pi [1655].

**James Gregory [Gregory, James]**

mathematician/astronomer

Scotland

1671

He lived 1638 to 1675 and invented expressions for pi [1671].

**Abraham de Moivre [Moivre, Abraham de]**

mathematician

France/England

1718 to 1722

Doctrine of Chances [1718]

He lived 1667 to 1754 and invented DeMoivre's theorem [1722].

**Christian Goldbach [Goldbach, Christian]**

mathematician

Germany

1742

He lived 1690 to 1764. All even integers greater than 2 are sums of two primes {Goldbach's hypothesis, Goldbach} [1742].

**Caspar Wessel [Wessel, Caspar]**

mathematician

Norway

1797

He lived 1745 to 1818 and placed complex numbers on a plane with two perpendicular coordinates.

**Jean Robert Argand [Argand, Jean Robert]**

mathematician

France

1806

He lived 1768 to 1822 and placed complex numbers on a plane with two perpendicular coordinates.

**Sophia Germain [Germain, Sophia] or Sophie Germain [Germain, Sophie]**

mathematician

France

1816 to 1820

She lived 1776 to 1831 and studied number theory and elasticity [1816]. For integers  $x$ ,  $y$ , and  $z$ , if  $x^5 + y^5 = z^5$ , then  $x$ ,  $y$ , or  $z$  must be divisible by 5 [1820] {Germain's theorem}.

**Hermann Grassmann [Grassmann, Hermann]**

mathematician

Germany

1844 to 1862

Theory of Linear Extension [1844]; Theory of Extension [1862]

He lived 1809 to 1877 and invented hypercomplex numbers {Grassmann variable}. Hypernumbers can represent tensors, quaternions, matrices, determinants, and all number types. Grassmann variables anti-commute:  $m \cdot n = -n \cdot m$ . He studied calculus of extension. Perhaps, space-time has extra, Grassmann dimensions to allow supersymmetry and supergravity.

**William R. Hamilton [Hamilton, William R.]**

mathematician

Ireland/Scotland

1850 to 1856

Lectures on Metaphysics and Logic [1869]

He lived 1788 to 1856 and belonged to school of intuition. He invented quaternions and Cayley-Hamilton theorem. He studied non-commutative algebras. His student was H.L. Mansel. People can know the finite, but people know the rest by faith, based on Kant.

**Richard Dedekind [Dedekind, Richard]**

mathematician

France

1870 to 1916

Essay on the Theory of Numbers [1870]

He lived 1831 to 1916 and studied fields, algebraic numbers, and irrational numbers.

**Leon Chwistek [Chwistek, Leon]**

mathematician

Krakow, Poland

1925 to 1935

Problem of Reality [1935]

He lived 1884 to 1937 and defined number.

**John Horton Conway [Conway, John Horton]**

mathematician

USA

1970

He lived 1937 to ? and invented an axiomatic number system, constructing counting numbers, and so all numbers, using rules for right and left. He invented Game of Life [1970], based on cellular automata.

**Yuri Matiyasevich [Matiyasevich, Yuri]**

mathematician

Russia

1970

He lived 1947 to ? and proved that no general algorithm can decide if Diophantine-equation systems have integer solutions, based on work by Julia Robertson, Martin Davis, and Hilary Putnam.

**Amir D. Aczel [Aczel, Amir D.]**

mathematician

USA

2000

Mystery of the Aleph [2000]

Aleph is symbol for infinity levels.

## **MATH>Mathematics>History>Series**

### **Guillaume de l'Hôpital [l'Hôpital, Guillaume de]**

mathematician

Paris, France

1696

Analysis of the Infinitely Small by Understanding Curved Lines [1696]

He lived 1661 to 1704 and studied series and invented L'Hospital's rules.

### **Brook Taylor [Taylor, Brook]**

mathematician

London, England

1715

Direct and Inverse Methods of Increments [1715]; Linear Perspective [1715]

He lived 1685 to 1731 and invented Taylor series and Taylor's theorem.

### **Colin Maclaurin [Maclaurin, Colin]**

mathematician

London, England

1720 to 1742

Organic Geometry [1720]; Treatise of Fluxions [1742]

He lived 1698 to 1746, invented Maclaurin series, and used determinants method to solve linear equations [1726 to 1729].

### **Thomas Simpson [Simpson, Thomas]**

mathematician

London, England

1745

Algebra [1745]

He lived 1710 to 1761 and invented Simpson's rule.

### **Joseph Fourier [Fourier, Joseph]**

mathematician

Paris, France

1822

Analytical Theory of Heat [1822]

He lived 1768 to 1830, invented heat equation, and invented Fourier series and Fourier transform: over intervals, any function can be trigonometric series.

## **MATH>Mathematics>History>Set Theory**

### **Bernard Bolzano [Bolzano, Bernard]**

mathematician

Bohemia

1837 to 1850

Scientific Theory [1837]; Paradoxes of Infinity [1850]

He lived 1781 to 1848 and studied continuity and set theory. Real numbers in closed intervals can be in one-to-one correspondence with real numbers in other closed intervals. Infinite sequences in closed intervals have limits {Bolzano-Weierstrass theorem}. Truths can be a priori. Logic is about ideals, not about time or space.

### **John Venn [Venn, John]**

mathematician

USA

1880

On the Diagrammatic and Mechanical Representation of Propositions and Reasonings [1880]  
He lived 1834 to 1923 and invented Venn diagrams.

**Georg Cantor [Cantor, Georg]**

mathematician  
Halle, Germany  
1885

Contributions to the Founding of the Theory of Transfinite Numbers [1885]

He lived 1845 to 1918 and studied set theory, infinity, continuity, transfinite numbers, union, intersection, conjunction, disjunction, bound, extension principle, abstraction principle, and one-to-one correspondence.

He invented continuum hypothesis. Cardinal-number series and ordinal-number series are infinite. Irrational numbers in closed intervals are rational-number-series limits. Sets of limits can have sets of limits, and so on, to infinity.

Geometrical-figure or space topologies are points related by distance functions or limits. For any real number  $n$ ,  $2^n > n$ .

**Cesare Burali-Forti [Burali-Forti, Cesare]**

mathematician  
Italy  
1897

Question about Transfinite Numbers [1897]

He lived 1861 to 1931. Ordinal numbers are well-ordered by definition. Ordinal-number sets must then have a greatest ordinal number. However, the set can be infinite and not have greatest ordinal number. Therefore, infinite ordinal-number sets cannot exist {Burali-Forti paradox}. Ordinal-number sets are higher-ordinal-number-set subsets.

**Ernst Zermelo [Zermelo, Ernst]**

mathematician  
Germany  
1904 to 1908

He lived 1871 to 1956 and invented Zermelo-Fraenkel set theory [1904 to 1908]. Infinite sets can contain sets with no elements in common. Methods to choose one element from each set must exist {axiom of choice, Zermelo}. If sets have no defined choice function, sets must use axiom of choice.

**Felix Hausdorff [Hausdorff, Felix]**

mathematician  
Germany  
1907 to 1919

Principles of Set Theory [1914]; Dimension and Outer Measure [1919]

He lived 1868 to 1942, invented generalized continuum hypothesis [1907], and invented Hausdorff space.

**Abraham Fraenkel [Fraenkel, Abraham] or Abraham Fränkel [Fränkel, Abraham]**

mathematician  
Germany  
1922

He lived 1891 to 1965 and improved Zermelo-Fraenkel set theory [1922].

**Paul J. Cohen [Cohen, Paul J.]**

mathematician  
USA  
1963 to 1966

Set Theory and the Continuum Hypothesis [1966]

He lived 1934 to ? and proved that continuum hypothesis was undecidable under set theory [1963].

**MATH>Mathematics>History>Statistics**

**Thomas Bayes [Bayes, Thomas]**

mathematician  
London, England  
1736 to 1761

Introduction to the Doctrine of Fluxions, and a Defence of the Mathematicians Against the Objections of the Author of The Analyst [1736]

He lived 1702 to 1761. Expected outcome is worth or gain multiplied by probability. Risk is expected-outcome divided by outcome value {Bayesian theory} [1761].

### **Epistemology**

Census, experimental, or statistical data can determine expected outcomes and find hypothesis probability {Bayesian confirmation theory}. Before evaluating new data, people already have beliefs about hypothesis risk and expected outcome. They know what they expect data to be if hypothesis is correct and what data happen no matter whether hypothesis is true or false.

### **Simeon-Denis Poisson [Poisson, Simeon-Denis]**

mathematician  
Paris, France  
1808 to 1837

On the inequalities of the methods of planet movements [1808]; On the movement of Earth's rotation [1809]; On the variation of arbitrary constants in mechanical questions [1809]; Researches in the probability of judgments of criminal and civil matters [1837]

He lived 1781 to 1840 and invented Poisson distribution.

### **Andrei A. Markov [Markov, Andrei A.]**

mathematician  
Russia  
1900 to 1913

He lived 1856 to 1922, invented probability theory using Chebyshev continued fraction [1900], and invented Markov process [1913].

### **Jerzy Neyman [Neyman, Jerzy]**

mathematician  
USA  
1933

He lived 1894 to 1981 and invented Neyman-Pearson hypothesis-testing theory [1933].

### **Karl Pearson [Pearson, Karl]**

mathematician  
USA  
1933

He lived 1857 to 1936 and invented Neyman-Pearson hypothesis-testing theory [1933].

### **Herman Chernoff [Chernoff, Herman]/Lincoln E. Moses [Moses, Lincoln E.]**

mathematician  
USA  
1959

Elementary Decision Theory [1959]

Chernoff lived 1923 to ?. Mises lived 1921 to ?.

### **Bruno de Finetti [Finetti, Bruno de]**

mathematician  
Italy  
1974

Theory of Probability [1974]

He lived 1906 to 1985. Probability is graded belief or judgments. If judgments are coherent and consistent, judgments converge to consensus with more data, and probability is relative frequencies observed in nature.

**MATH>Mathematics>History>Tensor**

**Gregorio Ricci-Curbastro [Ricci-Curbastro, Gregorio]**

mathematician

Italy

1894 to 1900

Methods of calculating absolute differentials and their applications [1900: with Levi-Civita]

He lived 1853 to 1925 and studied absolute differential calculus [1894]. He started tensors, spinors, invariance, covariant, contravariant, version orientation-entanglement relation, and bivector wedge product.

**Tullio Levi-Civita [Levi-Civita, Tullio]**

mathematician

Padua, Italy

1901

Note on the resistance of fluids [1901]

He lived 1873 to 1941 and studied tensors.

**MATH>Mathematics>History>Topology**

**Augustus Ferdinand Möbius [Möbius, Augustus Ferdinand]**

mathematician

Leipzig, Germany

1827 to 1837

Calculus of Centers of Gravity [1827]; Handbook on Statics [1837]

He lived 1790 to 1868, invented Möbius strip, and studied homogeneous coordinates.

**Maurice Frechet [Frechet, Maurice]**

mathematician

France

1906

Abstract Spaces [1928]

He lived 1878 to 1973 and studied function spaces and topology [1906], introducing metric spaces [1906].

**Simon K. Donaldson [Donaldson, Simon K.]**

mathematician

England

1983

He applied Yang-Mills gauge theory to four-dimensional manifolds [1983].