Point-Set Theory of Particles

Abstract

Specific abstract-space Ideas, corresponding exactly with specific hypercomplex-number-array Ideas, have physical-thing characteristics. Special hypercomplex-number-array Ideas define special abstract-space Ideas that define point sets and begin physical things. Point-set point numbers, configurations, and motions account for particle energy states and motions, quantum-mechanical waves, uncertainty principle, mass-energy densities, space curvatures, general relativity, and quantum mechanics. Point sets are both non-physical and physical. Point sets have abstract properties that come from non-physical special mathematical Ideas. Point sets have physical points and constraining boundaries required mathematically by physical-thing characteristics of special abstract-space Ideas and their hypercomplex-number-array Ideas.

Keywords

particle theory, quantum field theories, Standard Model, Yang-Mills gauge group, space-time, singularity, quanta, general relativity, quantum mechanics, string theory, quantum loop theory, point particles, abstract spaces, number arrays, point sets

1. Point-Set Theory of Particles

Particle theories must be consistent with the particle Standard Model and its Yang-Mills gauge group, as well as account for space-time and general relativity, energy quanta and quantum mechanics, and conservation laws.

2. Quantum Field Theories

General relativity has continuous fields with electromagnetic and gravitational waves. Quantum mechanics has discrete particles with energy quanta, quantum-mechanical waves, and transition matrices. Quantum field theories try to unify general-relativity fields and quantum-mechanics particles. For example, virtual-particle streams make continuous field lines.

In string theories and M-theory, particles are Planck-length one-dimensional strings or p-dimensional p-branes. In quantum-loop theories, particles are Planck-length two-dimensional quantum loops that make spin networks. In point-particle theories, particles are zero-length zero-dimensional points.

3. Strings

Strings have length or diameter and can have different lengths or diameters. Strings have internal rotations and vibrations.

3.1. Particle Properties

String rotations account for particle spins and orientations. Closed strings have two rotation modes and so can make spin-2 bosons (gravitons).

3.2. Space Dimensions

Special relativity and space conformal symmetry require that some particles have zero rest mass, so that they travel at light speed. Strings must have non-zero lengths and fundamental-frequency vibrations, so they cannot have zero rest mass by being only points or by having no motions. Zero-rest-mass-particle strings have string-vibration superpositions that cancel positive and negative components to make net vibration energy zero.

Because strings have many points and vibration modes, string-vibration superpositions can cancel only when space has a specific number of compactified dimensions, allowing special compactified-dimension configurations. Compactified dimensions have transverse vibrations both along axis and around axis, and these dual vibrations can cancel. More compactified dimensions increase positive energy. Fewer compactified dimensions increase negative energy. Four-dimensional space-time has no compactified dimensions, so in string theory it cannot have conformal symmetry {conformal anomaly}.

For bosonic string theory, conformal symmetry requires 26 space-time dimensions (22 compact space dimensions). For supersymmetric string theory, conformal symmetry requires 10 dimensions (6 compact space dimensions), because superstring theory has fermion-boson symmetries that cancel 16 dimensions. M-theory has supersymmetric string theories as subspaces, so conformal symmetry requires 11 dimensions (7 compact space dimensions).

Therefore, string theories require too many (and perhaps unobservable) dimensions.

3.3. Singularities

If particles are strings, particles have at least Planck length, so space cannot have point singularities.

3.4. Gauge Group

Strings have infinitely many points and many vibration modes. Therefore, strings allow many possible gauge groups, not just the Yang-Mills gauge group.

3.5. Fermion-Boson Unity

Bosonic string theory is a vector gauge theory that accounts for only open strings and spin-1 bosons. Supersymmetric string theory is a tensor gauge theory that accounts for open and closed strings and spin-1 and spin-2 bosons and has supersymmetry to unify fermions and bosons.

3.6. No Tachyons

Because strings have many points, and string-theories require compactified space dimensions, strings have complex-number vibration components, so particles can have negative (and positive) energies and masses. In special relativity, negative-mass particles travel faster than light speed. Bosonic string theory has negative mass particles. Superstring theory accounts for both fermions and bosons, and their interactions cancel any negative masses, so superstring theory has only positive-mass particles.

4. Quantum Loops

Quantum loops have diameter and can have different diameters. Quantum loops have internal rotations and vibrations.

4.1. Particle Properties

Quantum-loop rotations account for particle spins and orientations. Quantum loops can have two rotation modes and so can make spin-1 and spin-2 bosons.

4.2. Space Dimensions

Spin-network spins define three space directions and so define four-dimensional space-time. Therefore, quantum-loop theories do not require compactified dimensions.

4.3. Singularities

If particles are quantum loops, particles have at least Planck-length diameter, so space cannot have point singularities.

4.4. Gauge Group

Quantum loops have infinitely many points and many vibration modes. Therefore, quantum loops allow many possible gauge groups, not just the Yang-Mills gauge group.

4.5. Fermion-Boson Unity

Quantum loops and spin networks can have supersymmetry and so unify fermions and bosons.

4.6. No Tachyons

Quantum-loop theory accounts for both fermions and bosons, and their interactions cancel any negative masses, so superstring theory has only positive-mass particles.

5. Point Particles

Point particles have zero diameter and do not change diameter. Points have no rotations or vibrations.

5.1. Space Dimensions

Point particles have only real-number properties, so point-particle theories do not require compactified dimensions to make zero-rest-mass particles.

5.2. Singularities

If particles are points, space can have point singularities, violating space-time field continuity.

5.3. Particle Properties

Point particles have no mechanism for particle spin or orientation, and so no mechanism to make spin-2 bosons (gravitons).

5.4. Gauge Group

Point particles can have zero rest mass and account for all Standard-Model particles. However, point-particle theory has no mechanism to account for the Standard-Model Yang-Mills gauge group.

5.5. Fermion-Boson Unity

Point particles have no mechanism for unifying bosons and fermions.

5.6. No Tachyons

Points have no rotations or vibrations, and so no complex-number vibration modes, so point particles have only positive energies and masses.

6. Enlarged Points

Enlarging points makes three dimensions and Planck-length-multiple diameters. However, enlarged points have no parts or structure inside, so they are essentially the same as point particles.

7. Point Groups

Putting points together in compact groups makes three dimensions and Planck-length-multiple diameters. Point groups have more than one point and so parts and structures. If points have no forces or tension among them, point groups are essentially the same as point particles. If points have forces or tension among them, point groups are essentially the same as strings or p-branes.

8. Toruses

Tori have inside radius, outside radius, and cross-section, making three dimensions.

Reducing torus radius to zero makes a two-dimensional cross-section. If cross-section has Planck-length-multiple diameter and internal tension, zero-radius-torus particles are essentially the same as 2-branes. With no internal tension, zero-radius-torus particles are essentially the same as point particles.

Reducing torus cross-section to zero makes a one-dimensional radius. If radius has Planck-length and internal tension, zero-cross-section-torus particles are essentially the same as closed strings. With no internal tension, zero-cross-section-torus particles are essentially the same as point particles.

9. Geometric Fractals

Fragmenting straight or curved line segments into a series of separated shorter line segments makes dimension greater-than-zero and less-than-one. If line-segment lengths are Planck-length multiples and line segments have internal tension, fractal segments are essentially the same as open strings. With no internal tension, fractal segments are essentially the same as point particles.

Perhaps, fractals can overlap and interact to fill in missing points and make continuous whole-number space dimensions.

10. Vectors

Giving line segments direction and orientation makes one-dimensional directed line segments in three-dimensional space. Vectors involve only real-number energies and spins. Bivectors, trivectors, and so on, combine vectors. If they have no tension (and no vibration components), vector particles are essentially the same as point particles. If they have tension (and vibration components), vector particles are essentially the same as directed open strings.

11. Quaternions and Octonions

Giving line segments direction and orientation and adding a scalar makes one-dimensional directed line segments in multi-dimensional abstract space. Quaternions and octonions involve complex-number energies and spins. If they have no tension (and no vibration components), quaternion and octonion particles are essentially the same as point particles. If they have tension (and vibration components), quaternion and octonion particles are essentially the same as directed open strings.

12. Spinors

Giving line segments direction, orientation, and rotation makes one-dimensional rotating directed line segments in three-dimensional space. Spinors involve complex-number energies, spins, and orientations. Non-commutative spinors have non-symmetric opposite orientations.

Relativistic electron-spin theory (Dirac) and supersymmetric-string theory use spinors. Relativistic quantum-mechanics wavefunctions use spinor waves.

Bispinors are bivectors involving hypercomplex numbers. Bispinors (and antisymmetric tensors) can represent quark-antiquark pairs, such as pions and other bosons, because their four components have parity-violating gauge group SO(1,3) for relativistic half-integer-spin quantum fields.

Trispinors can represent three-quark particles, such as protons and other fermions, because their eight components have gauge group SO(3) for electroweak interactions and strong-nuclear-force interactions (Weinberg) (Salam).

However, spinor and twistor theories have inconsistencies with general relativity [Penrose, 2004].

13. Point Sets

Point sets {Point-Set Theory of Particles} are one or more separated points inside a Planck-length-diameter 3-sphere boundary. Rather than one point, point sets have any number of separate points (points are not a compact group). Rather than two endpoints connected by a string, point sets have points but no string, no string tension, and no string-vibration modes. Point sets can have different Planck-length-multiple diameters. Point sets have internal translations, vibrations, and rotations. Point-set points are somewhat like M-theory string endpoints confined to branes.

Point sets are intermediate among points, strings, and quantum loops, so they have the good results of point-particle, string, and quantum-loop theories and do not have the bad results. Like point-particle, string, and quantum-loop theories, point sets account for particles, particle properties, particle motions, quanta, fields, space-time, quantum-mechanical waves and transition matrices, and general-relativity energy-density and curvature tensors. Point sets account for initial-universe "quantum foam" and later universe properties.

13.1. Constraints

Point-set point motion constraints are like strong nuclear force, which increases strength with distance, keeping quarks in a "bag".

13.2. Mini-Atoms

Point sets are not mini-atoms, with central point and orbiting points, because they do not have central forces.

13.3. Mini-Molecules

Point sets are not mini-molecules, with bonds between points, because they do not have radial forces.

13.4. Dimensions

Because points have zero dimensions, point sets have zero dimensions. However, point-set point configurations (and motions) make extension, direction, and orientation, as well as density, viscosity, and phase, and so define dimensions. Point-set structures and motions first define fractional dimensions from zero up to one dimension. Point-set dynamic dimensions account for "quantum foam".

Point sets combine and overlap to build up to three independent spatial dimensions and one time dimension, united in space-time. Perhaps, dynamic dimensions hybridize three space dimensions and one time dimension to make space-time. Perhaps, dynamic dimensions resonate three space dimensions and one time dimension to make quantum-mechanical waves in space-time.

Because they do not have strings, and so do not have tensions or too many points, point sets do not require compactified dimensions.

Perhaps, point-set interiors have no space-time and no metric.

13.5. Motions

Point-set points can have random motions, periodic chaotic orbits, and harmonic longitudinal and transverse vibrations/rotations. Internal motions have topological constraints. Point-set relative internal motions account for particle spins and orientations.

13.6. Vibration Components

Point sets have internal parts and structures and so have longitudinal and transverse complex-number-frequency vibrations along and across all space dimensions. Point configurations determine resonance frequencies and make point-set particles have only positive energies and masses.

Point sets must have more than one point and so cannot have zero diameter. However, because they do not have tension, point-set points can have no net vibration, so point-set particles can have zero rest mass without using compactified dimensions.

13.7. Quanta and Quantum Mechanics

Point-set vibration-wave equations have harmonic-frequency wavefunction solutions. Discrete wave frequencies represent energy quanta, which account for particle masses. Point-set wavefunctions are essentially the same as particle quantum-mechanical wavefunctions.

Point-set particle distributions determine total system energy.

13.8. Quantum Mechanics and General Relativity

Point sets can combine, overlap, and interact (like superposed wavefunctions) to make continuous fields and other structures. Point-set virtual-particle streams make continuous field lines. Point-set combinations can be both quantized and continuous and so unite quantum mechanics and general relativity.

13.9. Gauge Group

Similar to string theories, point-set theory accounts for all Standard-Model particles. Unlike strings with too many points, and single points with too few points, point sets can have just the right number of points, point structures, and motions to allow only one gauge group, the SU(3) x SU(2) x U(1) Yang-Mills gauge group. Like strings, point sets allow zero-rest-mass spin-2 bosons (gravitons).

13.10. Particle Masses

Like strings and quantum loops, point-set Planck-multiple-length diameters and constraining forces, which depend on electric charge and on point configurations and motions, determine vibration modes that account for particle energies and masses. Shorter diameters correspond to higher frequencies and energies. Point sets can have no net vibration energy, so particles can have zero-point lowest-energy state and zero rest mass.

Mass and energy are scalars and are always positive. Anti-mass and anti-energy are scalars and are always positive, but have opposite electric charge.

13.11. Particle Electric Charges

Positive charge comes from directed-point-set orientation. Negative charge comes from opposite directed-point-set orientation. Electric anti-charge is exactly opposite charge.

13.12. Particle Spins

Point-set points rotate around center or two-point axes. Point-set axes have two opposite orientations that represent clockwise and counterclockwise spins. Point sets can have more than one rotation mode.

Spin comes from clockwise or counterclockwise transverse-wave amplitude-vector rotation around oriented-point-set long axis. Boson spin 0 comes from two opposite-orientation spins. Boson spin 1 comes from two same-orientation spins. Graviton spin 2 comes from two perpendicular-orientation spins, one around each tensor axis. Fermion half-integer spins come from three clockwise or counterclockwise spins.

13.13. Particle Color Charges

Point-set complex-number oriented-point axes can be three vectors that add to zero, corresponding to an equiangular triangle. Anti-colors have opposite electric charge and opposite color charge.

13.14. Particle Strangeness

Point-set points can have configurations that represent non-opposite directions and orientations, so a pattern (strangeness) is present or absent, allowing parity or no parity. Anti-strangeness is pattern absence.

13.15. Structures

Point sets can have two points separated by Planck-length multiples. Two points with unsynchronized motions are like open strings. Two points with synchronized motions are like closed strings. Two-point point sets have shape and symmetry.

Point sets can have three points separated by Planck-length multiples. Points can be at three of seven possible positions and can have synchronized or unsynchronized motions. Three-point point sets have shape and symmetries. See Figure 1.

For point sets with more than two points, distances have ratios. For point sets with three points, square-line to vector-line ratio is 2:1, and triangle-line to square-line ratio is 3:2 (same as 4:3), so three-point point sets have harmonic lengths (and vibration frequencies).

Point sets do not have four points, because points have too many motion modes and too many gauge-group possibilities.

13.16. Particles

Point-set point configurations have symmetries that account for particles, exchange particles, forces, and conservation laws. Like strings and points, point sets can exist for real-particle lifetimes. Real-particle point sets have longer diameters, lower energies, and longer lifetimes.

13.17. Photons and Electrons

For U(1) photons, one point is at center point, second point has orientation away from center point, and third point is along same line in opposite direction, plus the points have unsynchronized motions. Photon three-point point sets make vectors with spin 1, no charge, and zero rest mass. Photons have photons as antiparticles, so both have the same three-point point sets.

Electron three-point point sets have spin 1/2, one negative electric charge, and non-zero rest mass. Electrons have positrons as antiparticles, so positron three-point point sets have spin 1/2, exactly opposite positive electric charge, and non-zero rest mass.

13.18. Intermediate Vector Bosons and Pions

For SU(2) intermediate vector bosons, one point is at center point, second point has orientation away from center point, and third point has perpendicular orientation to center-point and second-point line, plus the points have unsynchronized motions. Intermediate-vector-boson three-point point sets are vectors with spin 1, charge or no charge, strangeness, and rest mass. Intermediate-vector-boson antiparticles have spin 1, opposite charge, strangeness, and same rest mass.

Pion three-point point sets are vectors with half-integer spins, charge, strangeness, and rest mass. Pion antiparticles have half-integer spins, opposite charge, strangeness, and same rest mass.

13.19. Gluons and Quarks

For SU(3) gluons, three points are not at center and have orientations to center point that differ by 120 degrees, so the three points form an equilateral triangle around center point, plus the points have unsynchronized motions. Gluon three-point point sets are vectors with spin 1, no electric charge, color charge, and rest mass. Gluon antiparticles have spin 1, no electric charge, opposite color charge, and same rest mass.

Quark three-point point sets are vectors with half-integer spins, electric charge, color charge, and rest mass. Anti-quark three-point point sets are vectors with half-integer spins, opposite electric charge, opposite color charge, and rest mass.

13.20. Gravitons

For gravitons, three points are not at center and have orientations to center point that differ by 120 degrees, so the three points form an equilateral triangle around center point, plus the points have synchronized motions, making a circle. Flipping the circle makes the same figure, so figure has spin 2. See Figure 1. Graviton three-point point sets are single-symmetry tensors with spin 2, no electric charge, and zero rest mass. Gravitons have gravitons as antiparticles, so both have the same three-point point sets.

13.21. Other Bosons

Dilaton two-point point sets are scalars with spin 0. Axion three-point point sets are antisymmetric tensors with spin 0. Zero-rest-mass un-oriented point sets are SO(n) or Sp(n) bosons.

13.22. Uncertainty Principle

Point sets represent relations and can represent relation uncertainties. For spatial dimensions, point sets represent positions, position relations, and position uncertainties.

13.23. Virtual Particles

At small distances, the energy-time uncertainty principle allows virtual particles. Like strings and points, point-set pairs can arise spontaneously from space vacuum and exist for times inversely proportional to energy before virtual-particle annihilation. Virtual-particle point sets have short diameters, high energies, and short lifetimes.

Point sets always change to point sets, never to no point sets, because, by uncertainty principle, zero-length point sets have infinite energy. One zero-point-energy point set can become two virtual-particle point sets (particle creation). Two virtual-particle point sets can become one zero-point-energy point set (particle annihilation). The no-point-set (vacuum) state cannot exist. Point-set particle creations and annihilations preserve all symmetries and conservation laws.

13.24. Conformal Symmetry

Because point sets have no strings and so no tensions, point sets do not require compactified dimensions and can have conformal symmetry in four-dimensional space-time.

13.25. Density

Point sets can combine, overlap, and interact. Point-set combinations can make density, viscosity, and phase.

13.26. Fields

Point-set combinations can make scalar fields.

13.27. Boundary

Point-set point structures and motions define a boundary. At the boundary, point sets can combine, overlap, and interact to define new boundaries and change physical effects.

13.27. Space Curvature

Point-set combinations have boundary interactions that change point-set point structures and motions and make energy density and space curvature.

13.28. No Singularities

Point sets have at least Planck-length diameter, so space has no point singularities.

13.29. Dark Matter and Dark Energy

Perhaps, point-set points, point structures, and motions define dark matter and dark energy.

13.30. Microtextures

Point sets are nanoscopic three-dimensional structures. Point-set surface boundaries combine, overlap, and interact to make microscopic two-dimensional surface textures (microtextures).

13.31. Microtextures and Sensations

Perhaps, brain has pattern recognition of microtextures and interprets them as sensations.

13.32. Point Motions and Interactions

Particle points are Planck distance apart. Particle points move with Planck-time periods. Particle points have Planck energies. Particle points interact but do not have conventional forces.

Space at Planck distances is "quantum foam". Quantum-foam has Planck distances. Quantum-foam straight lines last less than Planck time. Quantum-foam has random kinetic energy.

Particle points move in "quantum foam". Point interactions always change distance and direction and so do not have separable transverse or longitudinal components. Point interactions oscillate but at random.

Point space-time positions and interactions define the point-set surface and its configuration. Points are always on the point-set surface. Point-set surfaces always change, and point interactions are along surfaces. Points can have different distances and interactions. For example, points can be equidistant or not. Points can have the same or different interactions. Point-set surfaces add structure to quantum foam.

Point interactions are attractive, neutral, or repulsive, depending on distance and point-set-surface configuration.

Points are discrete. Point interactions are continuous. Point sets are both discrete and continuous, so they can account for both quantum mechanics and general relativity.

Particles have three points, so they have unpredictable behavior, just like the gravitational three-body problem. Unpredictability accounts for quantum-mechanical uncertainties and probabilities.

In classical mechanics, momentum and position are independent, but in quantum mechanics, they are dependent. Particle points account for this.

14. Abstract Spaces

Abstract mathematical Ideas exist non-physically [Penrose, 2004] and include abstract-space Ideas.

Abstract spaces have states and trajectories that define point-set point numbers, configurations, and motions (which account for particle states and physical processes). Abstract spaces can change and have waves that define point-set quantum-mechanical waves that account for particle energies and motions and for uncertainty principle. Abstract spaces have densities and curvatures that define point-set densities and curvatures that account for general-relativity mass-energy densities and space curvatures. Abstract spaces are discrete but can overlap to make continuous structures, so point sets can unify quantum mechanics and general relativity.

15. Hypercomplex-Number Arrays

Abstract mathematical Ideas include hypercomplex-number-array Ideas.

Hypercomplex-number arrays represent abstract-space point numbers, configurations, and motions. Hypercomplex-number arrays can be quantum-mechanical transition matrices (with matrix mechanics) that account for abstract-space states and trajectories. Hypercomplex-number arrays can have waves that account for abstract-space quantum-mechanical waves. Hypercomplex-number arrays can be matrices that represent tensors that account for abstract-space densities and curvatures. Hypercomplex-number arrays are discrete but can overlap to make continuous structures, so abstract spaces can have discrete points and continuous fields.

16. Point Sets, Abstract Spaces, and Hypercomplex-Number Arrays

Specific abstract-space Ideas, corresponding exactly with specific hypercomplex-number-array Ideas, have physical-thing characteristics. Special hypercomplex-number-array Ideas define special abstract-space Ideas that define point sets and begin physical things. Point-set point numbers, configurations, and motions account for particle energy states and motions, quantum-mechanical waves, uncertainty principle, mass-energy densities, space curvatures, general relativity, and quantum mechanics.

Point sets are both non-physical and physical. Point sets have abstract properties that come from non-physical special mathematical Ideas. Point sets have physical points and constraining boundaries required mathematically by physical-thing characteristics of special abstract-space Ideas and their hypercomplex-number-array Ideas.

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